1975

Estimation of service life characteristics for the valuation of human resources

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BARTA, Thomas Arnold, 1935-
ESTIMATION OF SERVICE LIFE CHARACTERISTICS
FOR THE VALUATION OF HUMAN RESOURCES.

Iowa State University, Ph.D., 1975
Engineering, industrial

Estimation of service life characteristics for the valuation of human resources

by

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A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of

The Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Department: Industrial Engineering
Major: Engineering Valuation

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1975

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INTRODUCTION

It is recognized that people are valuable to an organization. The value of an organization is derived from the ability of the employees to render services that have economic value. It costs money to recruit and train a workforce to the point where it becomes a smoothly functioning team, and though it is generally accepted that people are an economic resource at least as important as mechanical equipment, few organizations attempt to account for their human resources. In spite of the comments in many annual reports that "people are our most important assets," it is futile to look for a valuation of these assets in the report. Conventional accounting systems treat investments in human resources as expenses rather than assets, so there is no encouragement for a manager to work toward an increase in the value of those human assets, even though it is known that this value can be appreciated or depleted as a result of the way people are managed.

In an effort to correct these inequities, attempts have been made to measure the value of an organization's human resources. During the past decade, there has been a growing interest in the idea of accounting for people as organizational resources (17). This interest has led to an emerging field of research known as "Human Resources

Accounting." Since there is a growing awareness of the importance of people to organizational effectiveness, the significance of study in this area can only increase in relevance, especially in employee-intensive organizations including industrial as well as universities, hospitals, athletic teams, and consulting firms.

LITERATURE REVIEW

Research has been concerned with answering questions such as those that follow. Shouldn't employees be treated at least as well as capital equipment, by providing for their maintenance and depreciation (9)? What is the value of the human resources in an organization (17) (18)? If a functioning organization had to start up tomorrow without any trained employees, what would be the cost (32)? What is the cost to the company of a terminated, quit, or transferred employee (1) (39) (40) (41)? Would there be any significant difference in management decisions if the value of human resources were known (12)?

Flamholtz (17) organized human resource research into three general categories: human resource value theory, measurement of human resource cost and value, and applications of human resource accounting in organizations. In relation to the theory, he said it is proceeding from two different directions. One as an outgrowth of studies on organization and leadership at the University of Michigan's Institute for Social Research (39) (40) (41). This is an attempt to develop a model of the determinants of a group's value to an organization. The other approach by Flamholtz (19) is an attempt to develop a model of the determinants of an individual's value.

The Michigan effort, under the leadership of Likert (32) has been directed toward a discovery of the organizational structure and the principles and methods of leadership and management which result in optimal organizational performance. He has formulated a model of the variables which determine the effectiveness of a human organization and has suggested that the model reflects the human resource value. Likert suggests it is probable that after sufficient research has been done and sufficient data and experience obtained, it will be feasible to do human asset accounting similar to the way that standard costs are now used to estimate the costs of new products and that MTM (Methods-Time-Measurement) is used to set a standard time for the performance of a task.

Pyle (6) (39) (40) (41), as Director of the Human Resource Accounting Program at the University of Michigan, has determined the costs of the human resources such as recruiting, hiring, training, experience and development and has applied his approach at the R. G. Barry Corporation. Human resource costs are identified and separated from other costs of the firm, then divided into categories such as recruiting, hiring, training and development. These costs are allocated among the managers according to the people supervised and rules have been set up for depreciating these assets over their expected useful life. With Pyle's assistance, the R. G. Barry Corporation 1969 annual report

(39) contained industry's first published financial statements to include human resource data. Both the balance sheet and income statement carried two columns of figures, one conventional and one that reflected the human resource investment.

Flamholtz (15) (16) (17) (18) (19) (20) has attempted to develop and assess the validity of a model of an individual's value to an organization. He selected the individual since measures of individual value can, in principle, be aggregated in order to value larger units of people, where the reverse may not be possible. He proposed a system where estimates are made of the time period during which a person is expected to render service to an organization, the service states which the person may occupy, the value derived by the organization if the individual occupies these states for a specified period of time and the probability that a person will occupy each state at specified future times. Stated more formally,

$$E(S) = S1P(S1) + S2P(S2) + S3P(S3) \ldots\ldots SnP(Sn)$$

$$= \sum_{i=1}^{n} SiP(Si)$$

where E(S) is the expected service, Si represents the quantity of services expected to be derived in each state and P(Si) is the probability that they will be obtained. He suggests that the system is essentially a stochastic process with rewards and defines a stochastic process as a natural system that changes in time in accordance with the laws of probability, where the rewards are the earnings of the system, and the state is the position currently occupied by the individual. According to Flamholtz, to measure an individual's value to an organization it is necessary to 1) estimate the time period during which the person is expected to render services to an organization since all values must be on a present worth basis; 2) identify the service states that the person may occupy; 3) measure the value expected to be derived from an individual if he occupies the state for a specified time period; and 4) estimate the probability that a person will occupy each state at specified future times. Since expected services are difficult to estimate, he suggests surrogate measures of individual value such as acquisition cost, replacement cost, performance appraisals, salary compensation or commissions. Sadan and Auerbach (42), along with Jaggi and Lau (26), have built on this approach and proposed stochastic models for the evaluation of human resources in an uncertain environment.

Hekimian and Jones (22) have suggested a system of competitive bidding for people within an organization to establish their value. Managers bid for any employee they need within the company and the bid is included in the winning bidder's investment center asset base. By placing a value on both the physical and human assets, it is possible to calculate a return on investment for each investment center. A successful low bid is the most desirable since the manager must carry the asset value in the investment base.

Lev and Schwartz (31) have proposed using discounted future compensation as a surrogate measure of human resource value. A person's value is the present worth of his remaining earnings from employment. The estimated human capital value of a person y years old is:

$$V_y = \sum_{t=y}^{T} \frac{I(t)}{(1+r)^{t-y}}$$

where $I(t)$ = annual earnings up to retirement

r = a discount rate

T = retirement age

To determine the total value of a firm's labor force, they suggest it be divided into groups such as skilled, unskilled, engineers, salesmen, etc. Average earnings profiles, based on census data are compiled for each group and the sum of the present value of each group provides the total human capital value. If census data is not typical of a particular firm, then earnings profiles based on the firms own wage scale can be constructed.

Operationally, all of the proposed strategies to human resource value contain problems yet to be solved. Pyle's approach suffers from a lack of definition concerning the costs to be included in the investment. For example, what part of the Personnel Department costs should be allocated to hiring? Is it legitimate to count the time spent by other managers in training a new man? Pyle, however, has a working system at the R. G. Barry Corporation and historical costs such as hiring and training have traditionally been used in financial statements since they are readily verified.

Flamholtz' model offers an overall approach to individual valuation, but his need to measure the value derived by an organization if an individual occupies a particular state for a specified period of time is difficult to fulfill since the determinants of value at this time have not been defined. Hekimian and Jones' system of competitive bidding is interesting, but probably not one that most managers would take seriously, since it has the flavor of game-playing. Lev and Schwartz' proposal of discounting future earnings is based on salary as a surrogate measure of value, whereas other measures might be more valid. The Likert model has had no test for validity as a set of determinants of the value of the human organization (17).

OBJECTIVES

In principle, the value of people to an organization is the present worth of the future services they are expected to render (17) (34). Currently, the development of human resource value theory is proceeding from at least two different directions (17), but expected service life is a common problem. Flamholtz stated (17), "To measure a resource value, it is necessary to forecast its expected service life (the valuation period) and estimate its expected future services. For a monetary valuation, expected future services must be translated into monetary terms and discounted to their present value."

The emphasis of this research then, was on the resource valuation period and expected service life. Specifically, the objectives were:

1. Investigate the applicability of industrial property life analysis techniques to the estimation of human resource service life.

2. Determine the extent that the Iowa survivor curves will adequately describe the service life characteristics of human resources.

3. Determine the nature of the survivor curves obtained from actual human resource retirement data.

4. Develop valuation models based on Engineering Valuation and Industrial Engineering concepts.

LIFE ANALYSIS

Insurance companies have long used the statistical method of studying human deaths to determine life expectancy and insurance premium rates (34). Survivor curves of human beings have been used for determining insurance rates for some 200 years, but only since 1902 have such curves been developed for physical property (34).

The basic idea of life analysis is to determine from historical records the dispersion of lives actually experienced in the past by the subject population, be it humans or physical property. The fundamental representation of this dispersion is a retirement frequency curve (Figure 1) which relates the number or percentage of retirements from some original placement to the property's age.

Observed retirement frequency curve data usually is very erratic and hard to analyze, so a cumulative form, the survivor curve (Figure 1), is more commonly utilized. The survivor curve indicates the percentage of an original placement of property that remains in service at ages zero to maximum life. The retirement frequency and survivor curves are related mathematically as follows:

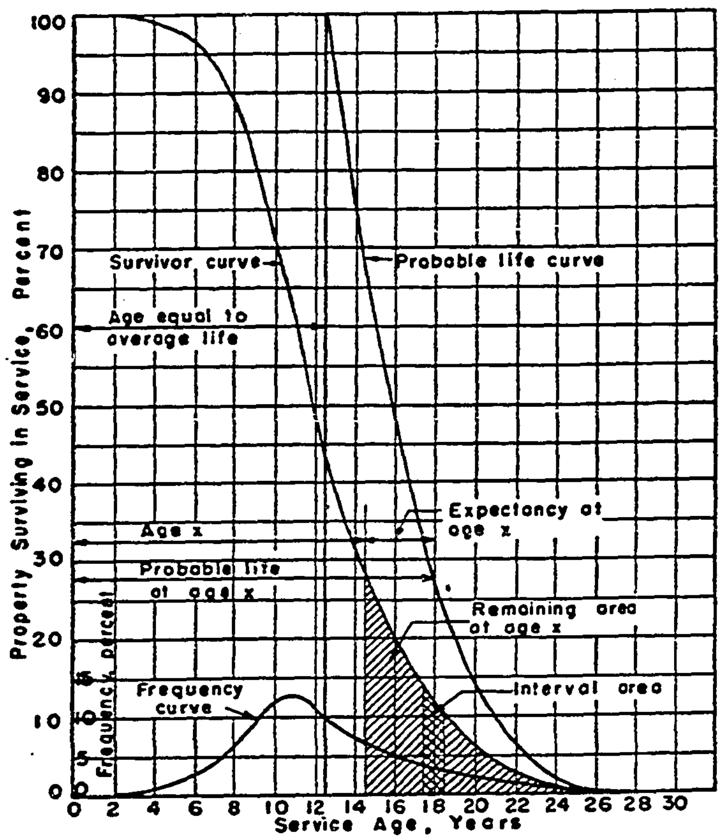

Figure 1. Survivor Curve (34)

Let y_i represent the decimal portion retired from an original placement in the i^{th} age interval. Then

$$\sum_{i=1}^{n} y_i = 1.0 \qquad (1)$$

where n is the age interval in which the last survivor of the placement is retired. The cumulative retirement fraction, Rx, from an original placement over the time span from age zero to the end of the x^{th} age interval would be

$$Rx = \begin{cases} \sum_{i=1}^{x} y_i & x = 1,2,3, \ldots, n \\ 0.0 & \text{if } x = 0 \\ 1.0 & \text{if } x = n \end{cases} \qquad (2)$$

Correspondingly, the portion of an original placement remaining in service at the end of the x^{th} age interval would be

$$Sx = \begin{cases} 1 - Rx & x = 1,2,3, \ldots, n \\ 1.0 & \text{if } x = 0 \\ 0.0 & \text{if } x = n \end{cases} \qquad (3)$$

which is the discrete version of the survivor curve.

Retirement dispersion can also be represented in terms of a series of retirement ratios for successive age intervals. A retirement ratic for an age interval is the amount of property retired during the age interval divided by the amount of property surviving at the beginning of the age interval, or

$$r_i = \frac{y_i}{S_{i-1}} \qquad (4)$$

and a survivor ratio for an age interval is defined as

$$s_i = 1 - r_i \qquad (5)$$

The retirement ratios and survivor ratios are related to the survivor curve expression as

$$Sx = So\prod_{i=1}^{x}(1-r_i) = So\prod_{i=1}^{x} s_i \qquad (6)$$

These relationships may also be shown in terms of continuous functions.

The primary use of the observed life dispersion pattern in any form is to measure life realized or to provide a basis for predicting remaining life. In valuation situations, remaining life is of interest since worth of remaining service is the objective. If the observed frequency curve is complete, that is, if it indicates the age at which every one of the original units was retired, a weighted average or mean life can be found. This life is known as the average service life in industrial property valuation situations. If this same dispersion pattern can be assumed to be a good prediction of how the retirements will occur in the future from a new property group at age zero, the mean life calculation is termed the probable average service life or the forecasted life of the average or typical property unit at age zero.

As mentioned above, the frequency curve form of dispersion data is hard to analyze so the survivor curve format is usually preferred. A complete curve would relate the percentage of the original property surviving from age zero to maximum age. The process of finding the weighted average of lives realized or to be realized described with respect to the frequency curve is analogous to finding the complete area under the survivor curve from age zero to maximum life and dividing this area by 100 percent, the percentage surviving at age zero. The area has the dimensions of percent-years, and dividing by 100 percent, the percentage of the property rendering the service, gives the length of service of the average or typical item, that is, the probable average service life in years. Life expectancy at any age is that period of time from the present age to the age when the unit will probably be retired from service. An estimate of expectancy for a typical unit at age x is determined by finding the remaining area under the forecasted survivor curve to the right of that age and dividing the amount by the percent surviving at age x. The probable life of an item or group of items of the same age is, by definition, equal to the age of the property plus the expectancy estimated as of that age.

The retirement frequency curve corresponds to a probability density function where the area under the curve must equal one. Integration of the curve over an interval is the probability of some original placement of property retiring during that age interval. Retirement ratios represent the conditional probabilities of retiring during an age interval, having survived to the beginning of the interval. A point on the retirement ratio curve represents the probability of retiring, having survived to that point in time.

Frequently, observed data which are to be utilized for making a life forecast are incomplete, that is, the age of the last retirement is not yet known, and the trend of the data is sometimes erratic. Plots of these incomplete data are called stub curves. Calculations giving estimates of average or remaining life need complete data, however, and are simplified if the trend of the curve is regular. Thus, observed data are normally smoothed and extended to maximum age prior to making life estimates.

A commonly used technique of smoothing and extending the survivor curve is to fit a polynomial to the series of observed retirement ratios by the method of least squares. A smoothed and complete survivor curve can be derived by inserting the retirement ratios determined for each age interval from the polynomial into equation (6). This amounts to starting with the amount installed and successively multiplying the amount surviving at the beginning of the age interval by one minus the retirement ratio for the age interval to obtain the amount surviving at the end of that age interval.

The survivor or frequency curves can be plotted to a scale in age as a percent of average service life, which gives a basis for classifying the curves by their basic mathematical shape. A study reported in 1935 by Robley Winfrey at Iowa State University (49) resulted in 18 type curves divided among three families, the left modal group of 6 curves, the symmetrical group of 7 curves, and the right modal group of 5 curves, the modes referring to the retirement frequency curves. Later on there were 4 more added corresponding to an exponential survivor curve, where the modal frequencies of the retirement frequency curve occur at or near the origin. Equations for these frequency curves are available (49) and will be useful in the remainder of this research.

These type curves are commonly referred to as the Iowa curves and are useful for at least two purposes:

1. To smooth original survivor curves and extend stub curves.
2. As an aid in determining probable life and life

expectancy.

Tables of these curves may be found in (49) and (34).

Plant property records in the case of industrial property, or personnel records in this research, are the source of data for studying retirement experience. Separate accounts or records may be kept for each individual unit or two or more units may be combined into a group, called a vintage group. Lamp (28) describes a complete property record as one that would permit determination of at least the following:

1. The amount of property installed each year (i.e., the amount installed each year as a vintage group).

2. The age at retirement of the property already retired from each vintage group.

3. The total amount of property in each vintage group surviving at the beginning of each year (plant balance of each vintage group at the beginning of each year).

Data that meet these requirements will be described as aged data in the ensuing discussion.

The type of property data available affects and sometimes determines the choice of methods for data analysis. Life analysis statistical methods are commonly divided into two categories which are dependent upon the available data:

turnover and actuarial. Actuarial methods include the retirement rate, individual unit, original group, composite original group and multiple original group, all giving probable average service life and the probable retirement dispersion pattern. The actuarial methods generally require a complete property record as described above. The turnover methods require data on the property installed and retired each year and the property balance each year. The simulated plant balance method is the only turnover method utilized in this research and it yields an estimate of probable average service along with a probable retirement dispersion pattern.

In practice, retirement data usually results in an incomplete survivor curve, that is, one that stops at a percent surviving greater than zero. This stub curve must be smoothed and extended to zero percent surviving before average service life is computed. Extension of the curves can be accomplished by judgement, statistical curve fitting, or matching to standard curves (34). The judgement method involves extending and smoothing the curve by eye along the most probable path. Statistical methods involve fitting equations to the data and include the Gompertz-Makeham, Weibull, and polynomial methods (23). Matching to standard or type curves involves use of a previously established set of type curves which are known to be representative in shape to those likely to be encountered. Some of these are the Iowa, Patterson, New York-h, and Gompertz-Makeham (14). The type curves are drawn on transparent paper and layed over the curve to be extended, which is drawn to the same scale. The stub curve is then extended to zero percent surviving along the path followed by the chosen type curve.

Henderson (23) found that the so-called graphical methods which have only a finite set of curves fit data as well as, if not better than the mathematical methods which fit an infinite set of curves.

VALUATION OF HUMAN RESOURCES

Several Engineering Valuation and Industrial Engineering concepts are useful in the valuation of human resources.

Engineering valuation

Marston, Winfrey and Hempstead (34) stated that the fundamental basis of the value of any specific property is the present worth, to the present owner, of the probable future services expected from the property during its probable future productive service life. They outlined the several approaches to the determination of property value. These can be compared with some of the recent efforts at human resource valuation.

1. The original cost of the property, adjusted for decreased usefulness and intangible elements. This represents the existing investment in the property and is similar Pyle's approach (39) in his work at the R. G. Barry Corporation, where the costs of recruiting, training and development have been determined and then depreciated over their expected useful life.

2. The replacement cost of the property, adjusted for decreased usefulness and intangible elements. Likert (32) has posed the question of the cost to start any existing organization from scratch with new, untrained people.

3. The earning value of the property. Past records of receipts and expenditures indicate probable future returns upon which to base the property value.

4. The service worth value of the property. The forecasting of future returns. Flamholtz (19) has proposed a model where the expected value of future services is estimated.

5. The market value of the property. Hekimian and Jones (22) have suggested a system of competitive bidding for people within an organization that would establish their value.

Engineering economy

Engineering economy has application when discounting future values to the present and in the consideration of a series that is changing by a constant amount or percent.

Present worth Values that lie in the future are usually discounted when they are expressed as of the current time. A given sum of money in hand today is worth more than the same sum to be received at some future date (34). Given an equal annual amount of money x for n years in the future at interest rate i, the present equivalent $p = x(\, p/a \,)_n^i$

(45). Given a future amount of money x, n years in the future, the present equivalent $p = x(p/f)_n^i$. Tables of values for $(p/a)_n^i$, $(p/f)_n^i$ and others are available (45). Any values established for human resources in the future should be discounted to the present time in this manner.

<u>Gradients</u> Salaries or future services are likely to increase or decrease in the future by either a constant amount, q, or a constant percent, r. Methods have been devised to find the present equivalent of a series behaving in this manner (45). The present worth of a gradient, g, is given by $p = g(p/g)_n^i$. The present worth of a constant percentage increase, r, may also be calculated. It can be shown that

when $r > i$

$$p = \frac{c}{(1+i)}\left[\frac{(1+w)^n - 1}{w}\right]$$

where $w = \frac{1+r}{1+i} - 1$

and c = the end of period amount that is changing by the constant percent r.

when $r < i$

$$p = \frac{c}{(1+r)} \left[\frac{(1+w)^{n} - 1}{w(1+w)^{n}} \right]$$

$$\text{where } w = \frac{1+i}{1+r} - 1$$

Markov chains

Flamholtz (19) provided a conceptualization of the individual valuation problem and called it a stochastic process with service rewards. He stated generally that

$$E(S) = S1P(S1) + S2P(S2) + S3P(S3) \ldots\ldots$$

or

$$E(S) = \sum_{i=1}^{n} SiP(Si)$$

where E(S) = expected service to be derived from an individual

Si = the quantity of services expected to be derived in each state or job

P(Si) = the probability that an individual will occupy this state in a future time period

Although this model has conceptual value, it lacks definition since the value, Si, depends on the length of time in the job. To provide computational tractability, the process can be defined as a Markov chain so that it will be possible to calculate the P(Si) and Si for fixed periods of time. A stochastic process is an indexed collection of random variables Xt, where the index t runs through a given set T (24). A Markov chain is a stochastic process which has the Markovian property. This Markovian property is equivalent to stating that the probability of any future event, given any past event and the present state $X_t = i$, is independent of the past event and depends only on the present state of the process (24). The conditional probabilities $P(X_{t+1} = j \mid X_t = i)$ are called transition probabilities and this notation is read as the probability of the random variable X being in state j at time t+1, given that it was in state i at time t. If, for each i and j,

$$P(X_{t+1} = j \mid X_t = i) = P(X_1 = j \mid X_0 = i)$$

for all t = 0,1,...,

then the transition probabilities are said to be stationary and are denoted by p_{ij}. This means that the transition probabilities do not change in time. The notation $p_{ij}^{(n)}$ is the conditional probability that the random variable X, starting in state i will be in state j after exactly n steps or time units.

The $p_{ij}^{(n)}$ must satisfy the following properties,

$p_{ij}^{(n)} > 0$ for all i and j and n = 1,2, ...

$$\sum_{j=0}^{M} p_{ij}^{(n)} = 1 \quad \text{for all } i,\ n = 1,2,\ldots, \text{ and } M = \text{the total number of states}$$

The transition probabilities are conveniently denoted in matrix form as

$$P^{(n)} = \begin{bmatrix} p_{00}^{(n)} & \cdots & p_{0M}^{(n)} \\ \vdots & & \vdots \\ p_{M0}^{(n)} & \cdots & p_{MM}^{(n)} \end{bmatrix} \quad \text{for } n = 1,2,\ldots$$

A full definition of a finite state Markov chain then, is a stochastic process that has the following properties (24):

1. a finite number of states
2. the Markovian property
3. stationary transition probabilities
4. a set of initial probabilities, $P(X_0=i)$ for all i

The Chapman-Kolmogorov equations provide a method for computing any n-step transition matrix. It can be shown (24) that the matrix of n-step transition probabilities is obtained from the expression

$$P^{(n)} = P\cdot P\ldots P = P^n = PP^{n-1} = P^{n-1}P.$$

So the n-step transition probability matrix can be found by computing the n^{th} power of the one-step transition matrix.

Given the initial one-step transition matrix P, it is possible to calculate any n-step transition matrix. This means the probability of being in any particular state 0 to M after any step, n, can be readily calculated.

Since it could be pertinent to human resource value application, the steady state condition of Markov chains should be considered. If states i and j are ergodic and belong to one class, then it can be shown that the probability of finding the process in a certain state j after a large number of transitions tends to the value π_j, independent of the initial probability distribution defined over the states. To show that all states belong to one class, that is communicate with each other, and are recurrent, that is

$$\sum_{n=1}^{\infty} p_{ii}^{(n)} \text{ diverges,}$$

it is sufficient to show that there exists a value of n for which $p_{ij}^{(n)} > 0$ for all i and j. Ergodic states are recurrent states that meet the additional condition of having two consecutive numbers, s and (s+1), such that the process can be in state i at times s and (s+1). If these conditions are met, then in the notation of Hillier and Lieberman (24),

$$\lim_{n \to \infty} p_{ij}^{(n)} = \pi_j$$

where the π_j's satisfy the following steady state equations:

1. $\Pi_j > 0$

2. $\Pi_j = \sum_{i=1}^{M} \Pi_i p_{ij}$ for $j = 1, 2, \ldots M$

3. $\sum_{j=1}^{M} \Pi_j = 1$

Substituting values for p_{ij} into these equations and solving simultaneously provides solutions to the Π_j's, the steady state probabilities.

If the random variables, X_t, represent different jobs in an organization, the transition probabilities, $P^{(n)}$, represent the conditional probabilities of going from job i to job j after n periods, assuming that the employee will still be employed during period n+1, and a value, S, is assigned to the quantity of services expected to be derived in each job over a year's time t, it would be possible to calculate the present worth of expected service, E(S), to be derived from an individual during his expected service life.

$$E(S) = S_{Start} + \sum_{n=1}^{L} \sum_{j=1}^{M} S_j P_{ij}^{(n)}$$

for all starting jobs, i,
M = total number of jobs,
L = life expectancy - 1

The transition matrix, P, must be established by an organization from an examination of past records, adjusted for the future. Since stationary transition probabilities are assumed here, the p_{ij}'s must be averages of the probabilities of job changes including employees in all time stages of a particular job. Unfortunately, the data for this research was in a form that made it inconvenient to extract information on job changes.

The value for n, job life expectancy of an employee, can be established by life analysis techniques to be outlined later in this research. S_j, the quantity of service expected to be derived in each state or job over a year's time, is the worth of probable future services expected from that job. It is an estimate of the contribution of each job to the total quantity of services provided by the enterprise. Sadan and Auerbach (42), proposed another Markov solution to human resources valuation, where the expected contribution, Sj, was assumed to be salary since most decision-makers acquire production factors whose costs do not exceed their contributions. Each state contained an n-tuple signifying some significant classification information about the employee in the state such as age and salary. Jaggi and Lau (26) proposed a similar Markov solution except they included an additional row and column in the transition matrix for the probability of leaving the company and then continued the iterations to steady state.

Most of the Markov solutions discussed up to this point have used identical transition probabilities to govern successive transitions. However, it is obvious that for any individual or group of employees, the transition probabilities will be non-stationary (25) since the probability of moving from one job to another is to some extent dependent upon the number of periods spent in the present job. An alternative and more realistic approach then would be to allow transition probabilities that can change from one time period to the next. Since the transition probabilities can change at each transition, there is little meaning to the idea of steady-state probabilities in most cases. The n-step transition probabilities are found by multiplying together, in order, the transition probability matrices for each of the steps.

If the transition probabilities are denoted in matrix form as before

$$\underset{\sim}{P}^{(n)} = \begin{bmatrix} p_{OO}^{(n)} & \cdots & p_{OM}^{(n)} \\ \vdots & & \vdots \\ p_{MO}^{(n)} & \cdots & p_{MM}^{(n)} \end{bmatrix} \quad \text{for } n = 1,2,\ldots$$

except another column and row are added to denote the state representing exit of the employees from the firm, then a situation exists where life analysis would again be helpful.

Since it would be expected, for example, that $p_{oo}^{(1)} > p_{oo}^{(2)}$ but at some point $p_{oo}^{(n)} < p_{oo}^{(n+1)}$ the situation is analogous to the conditional probability of moving from job i to job j, after having survived in job i for n periods, until at some point the employee would exit from the firm and then $p_{o\,exit} = 1$. At some point in time depending on the company and job, the transition probability matrix would be expected to arrive at

$$P^{(n)} = \begin{bmatrix} 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 1 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

The conditional probabilities are exactly the situation represented by the retirement ratio curve (Figure 5) and the "bathtub" shape is also typical of physical property. The retirement ratio curves are the basis for establishing the non-stationary transition probabilities.

EXPERIMENTAL PROCEDURE

Retirement characteristics of the personnel data provided by three large organizations were determined in several ways.

General

The applicability of industrial property life analysis techniques to the estimation of human resource service lives was tested by gathering actual employee data from the personnel departments of three organizations. Previously proven life analysis procedures were utilized in testing this data for average service life, the probable retirement dispersion pattern and an estimate of any discernible trends in the average service life. By actual use then, the applicability was shown.

The extent that the Iowa survivor curves adequately describe the service life characteristics of human resources was determined by a two step procedure. First the best-fitting Iowa curve was found by either plotting the experimental survivor data and overlaying the actual Iowa curves to provide a visual fit or using computer programs Actiput, and Selec to fit the best polynomial to the retirement ratio curve, convert the polynomial expression to it's equivalent survivor curve, and find the best-fitting Iowa curve in a least squares sense. Second, the retirements predicted by the best-fitting Iowa curve were statistically tested against the actual retirements to determine the goodness of fit.

The nature of the survivor curves obtained from actual human resource retirement data, that is, any generalization that can be made concerning the type of Iowa curve most applicable, was determined in the process of comparing the actual data with the Iowa curves. The negative exponential function was also investigated as a possible human resource survivor curve model. Average service lives for the employee groups included in the experimental data were a by-product of the comparison between the Iowa and actual survivor curves.

The majority of this research concerned an analysis of employee retirement and separation data provided by three large organizations in the insurance, manufacturing and public utility business. The types of employees covered were marketing, blue collar, career college graduates, engineers, mature females and clerical. The data itself showed the length of time employees stay with an organization, from the time they enter to the time of separation, whatever the reason; retirement, quit, fired, leave of absence, or laid off.

The retirement analysis was accomplished by three different methods, depending in part on the form of the data as it was supplied by the organization.

1. Retirement rate.

2. Original group.

3. Simulated plant balance.

In addition, some attempt was made at mathematical curve fitting, although this has been adequately researched elsewhere (23).

Retirement rate

If aged data is available, this method of calculating survivor curves is much the best since it is based on the collection and compilation of the data of all property in service during a period of recent years, both property retired and that still in service (34). It involves sampling retirement ratios, the number of people retired during the age interval divided by the number of people surviving at the beginning of the age interval, from the various vintages that have property still surviving during the period of study. A better understanding of this and the original group method can be gained by looking at Table 1 which shows the form of the data that is input to several computer programs for analysis.

An experience band shows the experience or retirements that have occurred during a band of years or a single year. For instance, referring to Table 1, in 1968 there were 150 units in service at the beginning of the year, and 30 of these retired during the year. The retirement ratio in this instance is 30/150 = .20. Of the 30 retired units, 7 were 0 to .5 years old, 6 were .5 to 1.5 years old, 7 were 1.5 to 2.5, 4 were 2.5 to 3.5, etc. The half years arise because the original data shows the number of people hired during a particular calendar year. The first period with the company for a group then, is assumed to be a half year since some were hired before, and some after, midyear. In terms of people, the experience band will show the retirement experience of an organization during a particular year or band of years. How does the retirement experience of 1972 compare with 1962? What is the trend of retirements since 1962 that may help predict into the future? These are some of the questions that can be answered with a retirement rate analysis.

Several existing computer programs (13) were utilized in the analysis of retirement rate data and are described briefly here. Actiput inputs aged data to computer storage in a form that can be interpreted by the remaining two programs. Output from this program is shown in Table 1. Another program called Tren for Trended Average Service Life performs rolling band and shrinking band analysis over a specified number of years. The analysis is conducted by fitting retirement ratios with polynomials from the first to third degree. A sample of this output is shown in Table 2. The last program is called

Selec for Average Service Life and Dispersion Selection Program. The program conducts a series of up to five actuarial analysis by fitting a curve of retirement ratios with polynomials and then comparing the smoothed survivor curve resulting from this polynomial to the Iowa curves to determine the best fit in a least squares sense. The output consists of the Iowa dispersion best-fitting the data and average service life based on the smoothed curve as shown in Table 3.

Retirement rate analysis followed this procedure:

1. Draw stub survivor curves for experience bands 1970-72, 67-69, 64-66 etc. in the same three year bands for all of the data so the retirement characteristics can be compared between the different organizations.

2. Use computer programs Actiput, Tren, and Selec to fit the best polynomial to the retirement ratio curve, find the Iowa curve of best fit, determine the average service life and analyze the trend of average service lives. In those cases where the Iowa curves, as determined by the Selec program, did not fit the data, the curves were matched by eye to the Iowa curves and the horizontal scale expanded when necessary. If the Iowa curves still did not fit, then the stub survivor curve was smoothed and extended using judgement, keeping in mind that a maximum

Table 1. Historical arrangement of retirement data

HISTORICAL ARRANGEMENT OF MORTALITY DATA PAGE 1 OF 2
OFFICE1

YEAR	PLANT IN SERVICE JANUARY 1	RETIRED DURING YEAR	PLANT ADD. RETIRED FROM THESE ADDITIONS	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM 0.5	1.5	2.5	3.5	4.5	5.5	6.5
1961	0.		27.	0.	0.	0.	0.	0.	0.	0.
		4.	4.	0.	0.	0.	0.	0.	0.	0.
1962	23.		31.	23.	0.	0.	0.	0.	0.	0.
		7.	3.	4.	0.	0.	0.	0.	0.	0.
1963	47.		33.	28.	19.	0.	0.	0.	0.	0.
		11.	5.	4.	2.	0.	0.	0.	0.	0.
1964	69.		45.	28.	24.	17.	0.	0.	0.	0.
		15.	4.	6.	2.	3.	0.	0.	0.	0.
1965	99.		51.	41.	22.	22.	14.	0.	0.	0.
		24.	4.	13.	2.	2.	3.	0.	0.	0.
1966	126.		43.	47.	28.	20.	20.	11.	0.	0.
		30.	9.	14.	3.	2.	2.	0.	0.	0.
1967	139.		34.	34.	33.	25.	18.	16.	11.	0.
		23.	8.	3.	6.	0.	3.	2.	1.	0.
1968	150.		38.	26.	31.	27.	25.	15.	16.	10.
		30.	7.	6.	7.	4.	3.	1.	1.	1.
1969	155.		47.	31.	20.	24.	23.	22.	14.	15.
		21.	4.	6.	2.	5.	2.	0.	0.	2.
1970	184.		36.	43.	25.	18.	19.	21.	22.	14.
		43.	9.	15.	5.	0.	2.	4.	4.	2.
1971	177.		38.	27.	26.	20.	18.	17.	17.	18.
		26.	12.	5.	2.	6.	0.	0.	1.	0.
TOTAL EXPOSURES			423.	328.	230.	173.	137.	104.	80.	57.
TOTAL RETIREMENTS			69.	76.	31.	22.	15.	7.	7.	5.

Table 1. (continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA
OFFICE1

PAGE 2 OF 2

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	7.5	8.5	9.5
1961	0.	0.	0.
	0.	0.	0.
1962	0.	0.	0.
	0.	0.	0.
1963	0.	0.	0.
	0.	0.	0.
1964	0.	0.	0.
	0.	0.	0.
1965	0.	0.	0.
	0.	0.	0.
1966	0.	0.	0.
	0.	0.	0.
1967	0.	0.	0.
	0.	0.	0.
1968	0.	0.	0.
	0.	0.	0.
1969	9.	0.	0.
	0.	0.	0.
1970	13.	9.	0.
	1.	1.	0.
1971	12.	12.	8.
	0.	0.	0.
TOTAL EXP.	34.	21.	8.
TOTAL RET.	1.	1.	0.

Table 2. Output from TREN program

IOWA STATE UNIVERSITY
ACTUARIAL TREND ANALYSIS
OFFICE1

RETIREMENT BAND	INDICATED AVERAGE LIFE FIRST DEGREE	SECOND DEGREE	THIRD DEGREE	ACTUAL RETIREMENTS FITTED	INDICATED RETIREMENTS FITTED FIRST DEGREE	SECOND DEGREE	THIRD DEGREE
1961-1963	127.4	130.6	9.6	10.	10.	10.	10.
1962-1964	91.6	2.9	2.9	21.	21.	21.	21.
1963-1965	85.4	2.9	49.6	37.	37.	37.	37.
1964-1966	84.6	3.6	54.9	52.	52.	52.	52.
1965-1967	82.8	4.3	66.4	56.	56.	56.	56.
1966-1968	77.5	5.2	12.1	59.	59.	59.	59.
1967-1969	77.7	9.4	5.9	55.	55.	55.	55.
1968-1970	56.0	4.8	45.9	74.	74.	74.	74.
1969-1971	65.5	6.1	59.8	65.	65.	65.	65.
1961-1971	70.5	6.3	62.7	165.	165.	165.	165.
1962-1971	70.5	6.3	62.7	165.	165.	165.	165.
1963-1971	70.3	6.2	62.0	161.	161.	161.	161.
1964-1971	69.1	6.1	60.4	155.	155.	155.	155.
1965-1971	68.4	6.0	59.7	144.	144.	144.	144.
1966-1971	69.7	6.2	62.0	124.	124.	124.	124.
1967-1971	68.9	6.8	63.6	103.	103.	103.	103.
1968-1971	64.5	6.0	58.6	88.	88.	88.	88.
1969-1971	65.5	6.1	59.8	65.	65.	65.	65.
1970-1971	58.4	5.8	52.7	48.	48.	48.	48.
1971-1971	91.7	10.9	7.7	14.	15.	14.	14.

Table 3. Output from SELEC program

AVERAGE LIFE BY WEIGHTED RETIREMENT RATIO METHOD
RETIREMENT RATIOS FITTED BY SECOND DEGREE

OFFICE1

RETIREMENT BAND1964-1965 DISP. 03 AVE. LIFE 3.6

AGE AT BEGINNING OF INTERVAL	EXPOSURES	RETIREMENTS ACTUAL	RETIREMENTS INDICATED	RETIREMENT RATIOS-PERCENT ACTUAL	RETIREMENT RATIOS-PERCENT SMOOTHED	LIFE-TABLE PERCENT OBSERVED	LIFE-TABLE PERCENT SMOOTHED
0.0	139.	17.	30.	12.23	21.76	100.00	100.00
0.5	116.	33.	31.	28.45	26.80	97.77	78.24
1.5	74.	7.	11.	9.46	15.35	62.80	57.27
2.5	59.	7.	5.	11.86	9.14	56.86	48.48
3.5	34.	5.	3.	14.71	8.19	50.11	44.05
4.5	11.	0.	1.	0.0	12.50	42.74	40.44
5.5	0.	0.	0.	0.0	22.05	42.74	35.39
6.5	0.	0.	0.	0.0	36.87	42.74	27.58
7.5	0.	0.	0.	0.0	56.93	42.74	17.41
8.5	0.	0.	0.	0.0	82.25	42.74	7.50
9.5	0.	0.	0.	0.0	112.82	42.74	1.33
TOTAL		52.	52.				
10.5					148.64		-0.17

length of service from age 18 to 65 is 47 years. Then the area under the curve was determined graphically in order to calculate average service life. The 47 year restriction was also a factor in selecting the best Iowa curve with the Selec program. A description of this graphical curve fitting procedure is available (34).

3. Provide a visual comparison between the smoothed polynomial-fitted curve, the best-fitting Iowa curve, and the stub survivor curve.

4. Statistically test the retirements predicted by the best-fitting Iowa curve with the actual retirements to determine the goodness of fit.

Original groups

An original group or vintage group is that group of people hired during the same year, or in the case of a multiple original group, during several years, but considered as a single, common group. This type of analysis is of interest if it is desired to compare the group of people hired in 1950 with those hired during 1974 in terms of service life. Is the average service life of employees hired today much shorter or longer than it was ten years ago? Is the trend of recently hired college graduates toward longer or shorter service lives with the organization? Original group analysis will answer these questions.

The original group life study was done graphically, and the procedure follows:

1. Draw stub survivor curves for three year original groups and selected individual years.

2. Smooth and extend the stub survivor curves graphically. The Iowa type curves are available on transparent paper with the average service life predetermined. The type curves were layed over the stub and the stub was extended to zero percent surviving along the path followed by the best-fitting Iowa curve. When the Iowa curves did not fit the data, they were smoothed and extended by judgement graphically, and the average service life determined in a manner identical to that followed in the immediately preceding section on retirement rate procedure. This method of determining average service life is explained in the section on curve fitting and in reference (34).

3. Statistically test the retirements predicted by the best-fitting Iowa curve with the actual retirements to determine the goodness of fit.

Simulated plant balance method

This is a technique that provides an estimate of the retirement distribution and average service life of property units or people where the records kept do not provide the age at which the property units were retired from service. The simulated plant balance method is a way to estimate survivor characteristics using a method of successive approximations. Each approximation requires that an estimate of the survivor characteristic described by the combination of a type curve and average life be tested using the annual additions and year-end balances (14). The data required are several year-end book balances and the gross additions from which these book balances resulted. A brief description from Methods of Estimating Service Life and Depreciation (14) is included here.

> The calculated year-end balance is the sum of the simulated survivors from each vintage of additions. The survivors are calculated for each vintage by multiplying the addition in each year by the percent surviving corresponding to the age of each addition as of the date of the year-end balance to which the calculated balance is compared. Several year-end balances should be simulated in order to obtain an adequate sample from which to determine the best fitting survivor curve for the group. The survivor curve which produces the least sum of squared differences between actual balances and simulated balances is considered that survivor curve of those tested which best fits the stub survivor curve inherent in the group studied.

The importance of this method is considerable since it can be expected that many organizations, especially smaller ones, will not have the type of detailed, aged, personnel data necessary for the retirement rate or original group analysis.

STATISTICAL DESIGN

Of considerable interest was how close the standard Iowa curves came to describing the experimental data obtained from the retirements of employees in the several organizations that provided data. Use of an Iowa curve to describe retirement experience will save a great deal of time because of tables already available for each curve, showing percent surviving and probable life at each age along with the mathematical equation for the retirement frequency curve.

A computer program (Selec) was used to find the best-fitting Iowa curve to any experimental retirement rate data by the least-squares method. This program fits the best polynomial to the retirement ratio curve since a polynomial is more likely to fit a retirement ratio curve than a survivor curve. The survivor curve resulting from the retirement ratio curve is then compared with the standard Iowa curves to find the best fit in a least squares sense. Original group data was fit graphically to the Iowa curves.

This pre-selected Iowa curve then is of interest because it is the best of the Iowa curves, but how good? To determine the goodness of fit of the Iowa curve selected, a statistical test was designed to compare the retirements predicted from the selected curve with the actual observed retirements. Two slightly different procedures were followed depending on the method of analysis, original group or retirement rate, even though they both involve the chi-square test.

Original group

The chi-square test statistic

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - np_i)^2}{np_i} \qquad \text{degrees of freedom (df)} = k-1$$

was used where

f_i = frequency of retirements during an age interval

n = the number of people in the original group

p_i = expected percentage of retirements during the same age interval, taken from an Iowa curve

This test was appropriate since it is a goodness of fit test based on the discrepancy between observed frequencies of values and expected or theoretical frequencies and is applicable to either a completely or partially specified distribution. The observed frequencies came from the actual retirements while the theoretical frequencies were the result of multiplying n, the number of people in the original group, by p_i, the expected percentage of retirements predicted by an Iowa curve.

The data was grouped so that each expected frequency of retirements, np_i, was >5. The calculated χ^2 was then compared with the tabled values at the 10 percent and 1 percent levels to determine whether the Iowa curve described the retirement experience of the group tested.

Retirement rate

Since the experience of a single population is not followed, but retirements from each vintage that has property still surviving during the period of study are sampled, the data available for test was viewed as coming from k different populations. Persons in the kth population were those who remained with the company at least k-1 years. Then the probability that the person retired within the next year was denoted p_K. The hypothesis tested then was

$$
\begin{aligned}
H:\ p_0 &= p_0' \\
p_1 &= p_1' \\
&\ \vdots \\
p_K &= p_K'
\end{aligned}
$$

Where p_0 = Probability(retiring during year 0)

p_0' = Probability(retiring this year taken from an Iowa curve)

If H is true then

$$W = \sum_{j=0}^{k} \frac{(r_j - n_j p_j')^2}{n_j p_j' (1 - p_j')}$$

is distributed approximately as χ^2 with d.f. equal to the number of age groups.

n_j = number of people at year 0 that the jth retirements came from

r_j = retirements during year j

To show this, it is assumed that the personnel in the different populations are independent, thus the number of retirements, r_K, during the kth year are independent random variables, each with a binomial(n_K, p_K) distribution. Letting x be a binomially distributed variable then

$$\frac{(x - \mu_x)^2}{\sigma_x^2} \dot{\sim} \chi^2_{1df}$$

because if $x \sim$ Binomial(n,p),

$$\mu_x = np, \quad \sigma_x^2 = np(1 - p),$$

then $x \dot{\sim}$ Normal(μ_x, σ_x^2)

$$\therefore \frac{x - \mu_x}{\sigma_x} \dot{\sim} \text{Normal}(0,1)$$

$$\therefore \frac{(x - \mu_x)^2}{\sigma_x^2} \sim \chi^2_{1df}$$

The test statistic, W, is the result of summing over the populations since they are assumed to be independent. The test statistic, W, was utilized in a manner identical to the original group approach.

Application

Application of this statistical theory dictated the need for three additional computer programs to expedite the analysis.

Integration of retirement frequency curve

Calculation of the chi-square statistic required a knowledge of the theoretical probability of retiring corresponding to some best-fitting Iowa curve. These probabilities are shown as p_i and p_j' in the previous discussion. The most efficient manner of finding these was by integration of the retirement frequency curves that have been derived and are now the basis of the Iowa survivor curves. These equations are available (49) and since the Origin-modal types occurred most frequently in this research, are shown in Figure 2 along with the survivor curves. The equation for the Iowa, type O4, retirement frequency curve, for example, is

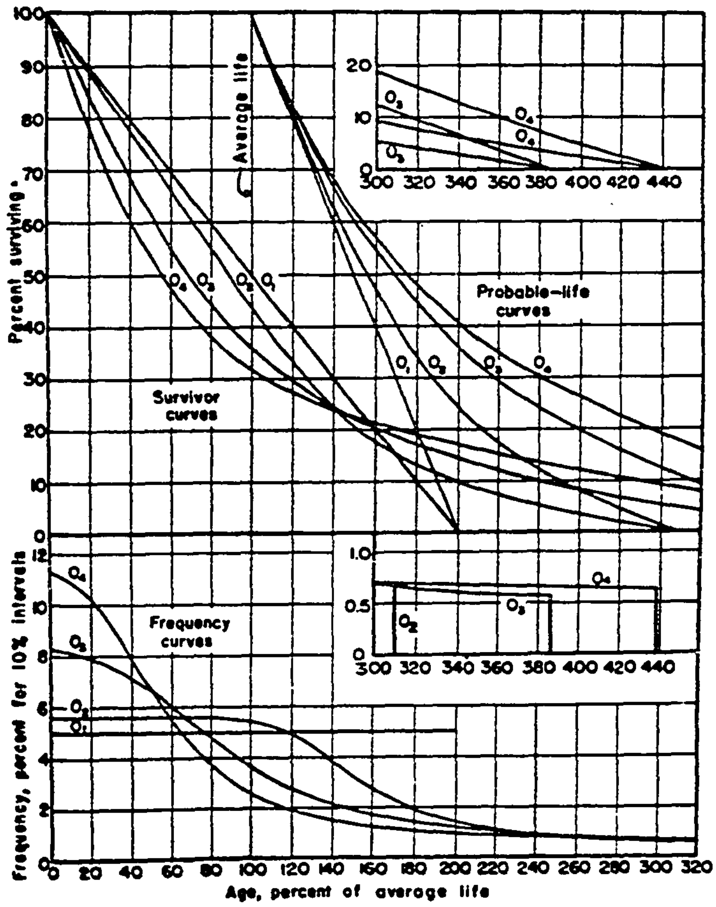

Figure 2. Survivor, probable life and frequency curves for the origin-modal type (49)

$$Y = 11\left[1 + (0.1x + 1.6)^6\right]^{-0.6} + .59863384$$

where x is the age in units of 10 percent of average life measured for all values of x from -10 to +34.0 and Y is the frequency in percent for 10 percent intervals.

The computer program utilizes an I.B.M. program called QATR found in the Scientific Subroutine Package. It performs integration of a given function by the trapezoidal rule after being given the upper and lower bounds of the interval to be integrated, the maximum number of bisections of the interval, the function to be integrated and the upper bound of acceptable error. Repeated trials of this routine showed it to be accurate to within one-half percent when integrating several intervals over the range of the function and comparing to 100 percent. Since the x values in the retirement frequency curve are in units of 10 percent of average life measured from the average life ordinate, an age of 0 gives an x value, assuming an 04 curve with 9.7 A.S.L., of

$$\frac{9.7}{.97} = -10.000.$$

At age 1,

$$x = \frac{9.7 - 1}{.97} = -8.969$$

The program listing and example of the output are in Appendix

A (Figure 11).

<u>Original group chi-square test program</u> The probabilities determined from the integration program were input to this program to calculate the expected number of retirements from the original group during successive age intervals. A sample of the output and program listing are shown in Appendix A (Figure 10).

<u>Retirement rate chi-square test program</u> This program is essentially identical to the original group chi-square program except for the summation statement, Appendix A (Figure 9). Since the retirement probabilities from the retirement frequency curves apply to original groups, it was necessary to be able to go back to the original groups that contributed the retirements during each age interval of the experience band. These original groups, n_j, are part of the input to the program, along with actual retirements, r_j, and the probabilities of retiring, p_j, that were output from the integration program.

EXPERIMENTAL DATA RESULTS

Summarized here are the experimental results of the retirement rate, original group, and simulated plant balances methods of life analysis along with the statistical comparison of actual retirements against retirements as predicted by the Iowa curves.

Retirement rate

A summary of the various retirement rate analysis by organization, showing the best-fitting Iowa dispersion and average service life is shown in Table 4.

Historical arrangement of the Manufacturing Marketing data, the basis for the various analysis, is shown in Appendix C (Table 9). Appendix B provides a visual comparison between the best-fitting Iowa curve, the actual stub curve and the polynomial-smoothed curve. On the basis of inspection only, the match was quite good. The dispersions for these Marketing employees (Table 4) were either O3 or O4 and the average service lives went from 9.0 to 14.9 years. A trend analysis of average service life in three year bands from 1941 to 1973, Appendix C (Table 10) showed no noticeable difference between service lives along this 32 year span.

Historical arrangement of the Office Career College Graduate data is shown in Table 1 of the previous section. Appendix B provides a visual comparison of the actual data, best-fitting Iowa curve, and smoothed polynomial-fitted curve. The summary shows that for this group of employees the dispersions were O1, O3 or O4 and the average service lives ranged from 3.6 to 9.8 years. A trend analysis of average service lives in three year bands from 1961-71 showed no trends toward increasing or decreasing lives over this time span.

Historical arrangement of the Utility Meter Readers is shown in Appendix C (Table 12). The original data from the Utility was somewhat unusual in that there was a record of those hired and separated between 1969 and 1973, while the record extended back to the year 1919 on those employees still employed as of 1969 to 1973. This gave an extended picture of the surviving employees by age, although a poor original group analysis since there was no record of the dates employees separated until 1969. Appendix B, Figures 49, 50, 51, shows the different survivor curve comparisons for the Meter Readers. Use of the Selec program for this group of employees indicated that the Iowa dispersions were all O4 and average service lives were 13.0, 14.7 and 16.6. It was obvious that these service lives were not accurate. The percent surviving dropped off so rapidly that a polynomial was unable to accurately fit the retirement ratios.

Table 4. Summary of retirement rate analysis

Experience Band	Manufact. Market.		Office Career College		Utility Meter R.		Utility Laborers	
	Iowa	ASL	Iowa	ASL	Iowa	ASL	Iowa	ASL
1955-57	C3	14.9						
1961-63	03	14.3	01	9.8				
1964-66	04	9.0	03	3.6				
1967-69	04	12.8	04	9.4				
1970-72	04	10.6			04	7.7[1]	04	.9[1]
1970-71			04	5.8				
1969-71					04	7.0[1]	04	.4[1]
1971-73					04	10.0[1]	04	1.9[1]

Experience Band	Utility Mechanics		Utility Ironworkers		Utility Coal-Ash		Utility All-Union	
	Iowa	ASL	Iowa	ASL	Iowa	ASL	Iowa	ASL
1970-72	04	6.1[1]	04	.5[1]	04	7.4	04	3.5[1]
1969-71	04	6.1[1]	L0	.8	04	2.1[1]	04	1.9[1]
1971-73	C1	9.9[1]	04	1.0[1]	04	9.3	04	4.5[1]

[1]Average service life determined graphically.

Since the program was written to match an Iowa curve to the survivor curve resulting from the retirement ratio polynomial, the best-fitting Iowa curve was also not accurate. Expanding the horizontal scale and plotting by eye to the standard Iowa curves was unsuccessful. Graphically determining the actual areas showed that the average service lives were 7.0 for 1969-71, 7.7 for 1970-72, and 10.0 for 1971-73.

Laborers, Mechanics, Ironworkers, Coal and Ash Handlers, and All Union Employees from the utility were analyzed in the same fashion as the other employee groups and the resulting historical arrangement, survivor curves and trend analysis is shown in Appendix B and C. In all cases where the polynomial-fitted curve was not accurate, average service life was determined graphically. Laborers and Ironworkers had average service lives on the order of one year or less. In most cases, over 90 percent of them left the same year they were hired. Since the trend analysis was based on a polynomial fit to the retirement ratios, and since this fit was not good, the trend analysis of this data was also not accurate. However, Table 4 indicates a lengthening of average service lives in recent years.

Original groups

Table 5 is a summary of the various original group analysis by job, showing the best fitting Iowa dispersions and average service lives. The data for the original group survivor curves can be duplicated by referring to Appendix C which contains the historical arrangement of retirement data.

For example, the Utility Meter Readers, Appendix C (Table 12), began with a group of 25 hired in 1969, of which 16 retired that same year. Following down the page at an angle to the right, this left 9, of which 5 retired, leaving 4 and so on.

Manufacturing Marketing personnel had average service lives ranging from 10 to 18 years and all of them showed a close conformance to the Iowa O4 type survivor curve. There was a definite trend to shorter service lives in more recent years as might be expected. The 1958-60 group had an 18 year A.S.L., 1961-63 a 14 year A.S.L., 1964-66 and 1967-69 a 10 year A.S.L.

The Office Career College Graduates exhibited the same behavior, beginning with a 10 year A.S.L. from 1961-63, 8 year A.S.L. for the original groups 1964-66 and 1967-69. They followed the Iowa O4 dispersion closely.

Since the placements of Utility personnel were over the short period of time 1969-73, there was not sufficient experience to obtain anywhere near complete survivor curves. However, using the data available, for Meter Readers the two original groups 1969-71 and 1970-72 showed an A.S.L. of 3 years after the stub curves were extended using an Iowa O4 dispersion.

Table 5. Summary of original group analysis

Original Group	Manufact. Market.		Office Career College		Utility Meter R.		Utility Laborers	
	Iowa	ASL	Iowa	ASL	Iowa	ASL	Iowa	ASL
1958-60	O4	18						
1961-63	O4	14	O4	10				
1964-66	O4	10	O4	8				
1967-69	O4	10	O4	8				
1969-71					O4	3	O4	.5
1970-72					O4	3	O4	.5

Original Group	Utility Mechanics		Utility Ironworkers		Utility Coal-Ash		Utility All-Union	
	Iowa	ASL	Iowa	ASL	Iowa	ASL	Iowa	ASL
1969-71	O4	5	O4	.5	O4	3		2.8
1970-72	C4	3	O4	.5	O4	4		3.9

Over half of these Meter Readers left the same year they were hired. In a like manner, the Utility Laborers, Mechanics, Ironworkers, Coal and Ash Handlers, and All Union Employees were plotted and fit to Iowa curves. The All Union Employee original groups differed from the Iowa curves to the extent that the actual data was extended by eye and the average service life found graphically.

Simulated plant balances

Some of the data was not aged, making it impossible to trace original groups or calculate retirement ratios. In some cases, only the number hired or terminated and the balance of personnel by year was available. This was true of the employee groups called Office Mature Females Over Age 30, All Home Office Employees, and Manufacturing Hourly. Determination of average service life and Iowa dispersion was accomplished by the Simulated Plant Balances method explained previously.

Additions, retirements and balances for Mature Females, Over Age 30 during the years 1957 to 1972 are shown in Appendix C (Table 24). Results of the analysis, Appendix C (Table 25), show that the best-fitting Iowa dispersion was an O4, with an average service life of 8.4 years. This best-fitting dispersion was determined by the minimum sum-of-squared difference between actual and simulated balances. The Index of Variation is another measure of agreement between actual and simulated balances and was calculated as follows:

$$\text{Index of Variation} = \frac{1000\sqrt{\dfrac{\text{Sum of Squared Differences}}{\text{Number of Test Years}}}}{\text{Average Actual Balance}}$$

The lower the value of the index, the better the agreement with the actual data since the actual and simulated balances should be as close as possible. The Retirement Experience Index is the portion of the oldest addition which would have been retired as of the date of the study had the property experienced the indicated dispersion and average service life. A value at or near 100 percent suggests at least a complete life cycle of the experience considered in the analysis.

Additions, retirements and balances for All Home Office Employees, Clerical and Other, Including Part-Time, 1948-72 are shown in Appendix C (Table 28). Results of the analysis showed that the best-fitting Iowa dispersion was an O4, with a 2.9 year average service life. An analysis using some of the same data for the years 1961-72 but excluding part time employees gave an R1 Iowa dispersion and a 3.2 year average service life. Removing the part time employees raised the average service life by .3 years. The overall short service life was due to the high turnover of young females in the clerical staff. A comparison of All Home Office Employees with Mature Females Over Age 30 showed that the average service life of Mature Females was more than twice that of the entire office.

Additions, retirements and balances for Manufacturing Hourly employees are shown in Appendix C (Table 30). Results of the analysis showed that the best-fitting Iowa dispersion was an L2, with a 4.7 year average service life. A summary of all Simulated Plant Balances analysis is shown in Table 6.

Table 6. Simulated plant balances summary

	Iowa Dispersion	Average Life(years)	Index of Variation
Office Mature Females Over Age 30	O4	8.4	91
Home Office Employees Clerical and Other Including Part Time	O4	2.9	55
Home Office Employees Clerical and Other Excluding Part Time	R1	3.2	37
Manufacturing Hourly	L2	4.7	154

Chi-square test results

The original or multiple original groups were chosen so they included sufficient years such that there were seldom less than 5 retirements during any one year. Also, the groups had to begin far enough back from the termination of the data to provide several years of experience. Where the retirement experience ended, the last age interval was broad enough to include and assume retirement of the remainder of the original group.

One year bands of data were picked for the retirement rate chi-square tests to satisfy the requirement of independence. Since it was necessary to know the number in the original group that the retirements in a particular year came from, the number of years included in the test was very limited for the Utility data. The original groups for these employees were known only from 1969 to 1973.

Results of the tests, Table 7, were generally favorable for all but the Utility employees when tested at the 10 percent and 1 percent levels. The values for $\chi^2_{.90}$ and $\chi^2_{.99}$ came from tables of percentiles of the chi-square distribution (33). These are the values at which there is a 10 percent and 1 percent probability of rejecting the hypothesis when it is true. This indicates that the Iowa curves described the retirement experience of the Manufacturing Marketing and Office Career College people well. In other words, the actual retirements came from a population described by an Iowa curve.

Table 7. Chi-square test results

	Mfg. Marketing			Office Career College			Utility Union Employees		
	calc χ^2	$\chi^2_{.90}$	$\chi^2_{.99}$	calc χ^2	$\chi^2_{.90}$	$\chi^2_{.99}$	calc χ^2	$\chi^2_{.90}$	$\chi^2_{.99}$
Original Group									
1961-64	14.33	16.0	23.2						
1964-67	19.72	12.0	18.5						
1967-70	8.37	7.7	13.3						
1941-51	21.54	32.0	41.6						
1952-62	17.14	18.5	26.2						
1961-63				11.71	14.7	21.7			
1964-66				12.62	10.6	16.8			
1967-69				12.21	6.2	11.3			
1969							926.99	9.2	15.1
1970							599.34	4.6	9.2
Retirement Rate									
1966	30.63	30.8	40.3						
1968	17.88	33.2	43.0						
1970	34.34	35.6	45.6						
1972	39.51	37.9	48.3						
1966				4.80	10.6	16.8			
1968				8.09	13.4	20.1			
1970				52.85	16.0	23.2			
1971							6222.22	6.2	11.3
1973							2402.34	9.2	15.1

The chi-square tests on the Utility Union employees were a formality only, since it was obvious from the survivor curves that the Iowa dispersions did not fit this data. In almost every case, over 80 percent of the employees left the first year, but the remainder stayed on for an extended period, some up to 46 years. There are no Iowa curves that presently fit this unusual retirement pattern. A more detailed report of actual and expected retirements, probabilities of retirement, number in original group and degrees of freedom is provided in Appendix C.

Curve fitting

The thrust of this research was the investigation of average service lives using previously accepted and proven techniques. Specifically, fitting polynomials to the retirement ratio curves, and when this failed, graphically finding the area under the survivor curve in order to calculate average service life. Other techniques have been investigated adequately by Henderson (23) and others with varying degrees of success. However, at least two obvious questions are raised when looking at the sharp slope of these curves during the early years and the fact that they all began at 100 percent. First, why not use a negative exponential function to describe the survivor curve, since it can be made to begin conveniently at 100 percent and the shape appears exponential? Second, why not fit a polynomial directly to the survivor curve rather than convert to a survivor curve after fitting to retirement ratios?

Several negative exponential functions were fit to a typical survivor curve in this research; the Office Career College Graduates, 1970-71 experience band. These curves are shown in Figure 3. The fit was not good, and in fact, a visual comparison between Figure 3 and Figure 23 was sufficient to show that at least in this instance, the best method was fitting a polynomial to the retirement ratio curve. Because of the general conformance to a survivor curve, however, the negative exponential could be useful in some cases for estimation purposes.

Another representative employee group, Office Career College Graduates, 1961-71 original group, was used to compare fitting a polynomial directly to the survivor curve or to the retirement ratio curve. To accomplish this, the actual survivor curve data was plotted, Figure 4, extended to 47 years, and the smoothed data used to calculate retirement ratios, Table 8. The resulting retirement ratio curve, Figure 5, and the survivor curve were fitted with polynomials using the program shown in Appendix A (Figure 12). This program fit the best polynomial in a least squares sense, and gave the X and Y values, along with the degree of polynomial requested. The best-fitting survivor curve polynomial was a third degree.

$$y = .83691 - .07472x + .00275x^2 - .00003x^3$$

The best-fitting retirement ratio curve was a second degree polynomial.

$$y = .24514 - .02877x + .00071x^2$$

Two additional CPS programs were written to assist in plotting the survivor curves from these points and are shown in Appendix A (Figures 13,14). One computed percent surviving for ages 0 to N in one year increments, given a polynomial expression for a retirement ratio curve. Negative retirement ratios were assumed to be zero. The other computed percent surviving for ages 0 to N in one year increments, given any polynomial expression.

Figure 5 shows the actual and smoothed retirement ratio curves. This "bathtub" shape is typical of that encountered in the failure of some physical property, notably electronics components. The second degree polynomial fit best. Figure 4 shows a comparison of the actual survivor curve, polynomial fit directly to survivor curve and polynomial fit to a retirement ratio curve. In this one example, there appeared to be little difference between the two methods of survivor curve fitting, except that the retirement ratio method can be made to begin at 100 percent.

Figure 4 and Table 8 also show the technique used to calculate graphically the area under a survivor curve, probable life and life expectancy. This method was followed in all cases where the Iowa curve fit was not satisfactory. Table 8 is a life table showing the percent surviving, retirement ratios, expectancy, probable life and provides a method of finding the area under the survivor curve and average service life by dividing the survivor curve into small rectangles. Column 6 is obtained by summing column 5, the interval area from the bottom upward to produce remaining areas. The area remaining at any service age divided by the percent surviving at that age results in the average expectancy of the surviving property, column 7. The probable life, column 8, of the survivors at each age is equal to their age plus their expectancy.

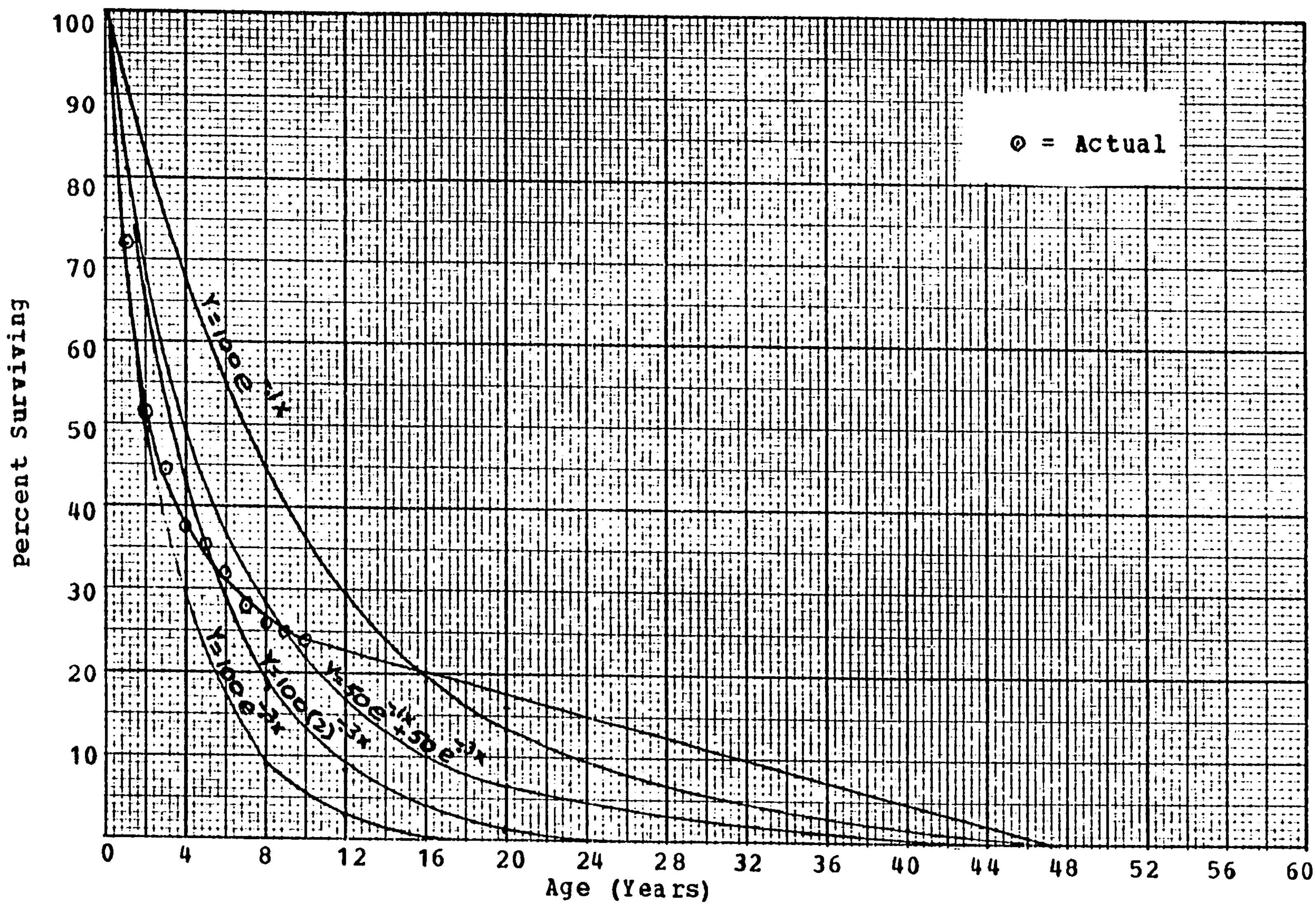

Figure 3. Exponential curve fitting to Office Career College Graduates, 1970-71 experience band

Table 8. Office Career College Graduates, 1961-71 original group

1[1]	2[1]	3[1]	4[1]	5[1]	6[1]	7[1]	8[1]
0-1	100.0	.1631	91.84	Same	1145.53	11.45	11.45
1-2	83.6	.2317	73.99	as	1053.69	12.59	13.59
2-3	64.3	.1347	59.97	col.	979.70	15.23	17.23
3-4	55.6	.1271	52.10	4	919.73	16.53	19.53
4-5	48.5	.1093	45.92		867.63	17.86	21.86
5-6	43.2	.0673	41.80		821.71	18.99	23.99
6-7	40.3	.0875	38.58		779.91	19.32	25.32
7-8	36.8	.0877	35.20		741.33	20.13	27.13
8-9	33.5	.0295	33.09		706.13	21.02	29.02
9-10	32.6	.0475	31.82		673.04	20.64	29.64
10-11	31.0	.0338	30.52		641.22	20.65	30.65
11-12	30.0	.0400	29.40		610.70	20.35	31.35
12-13	28.8	.0347	28.30		581.30	20.18	32.18
13-14	27.8	.0360	27.30		553.00	19.89	32.89
14-15	26.8	.0299	26.40		525.70	19.61	33.61
15-16	26.0	.0385	25.50		499.30	19.20	34.20
16-17	25.0	.0200	24.75		473.80	18.75	34.95
17-18	24.5	.0204	24.25		449.05	18.32	35.32
18-19	24.0	.0208	23.75		424.80	17.70	35.70
19-20	23.5	.0383	23.05		401.05	17.06	36.06
20-21	22.6	.0177	22.40		378.00	16.72	36.72
21-22	22.2	.0315	21.85		355.60	16.01	37.01
22-23	21.5	.0233	21.25		333.75	15.52	37.52
23-24	21.0	.0238	20.75		312.50	14.88	37.88
24-25	20.5	.0244	20.25		291.75	14.23	38.23
25-26	20.0	.0300	19.70		271.50	13.57	38.57
26-27	19.4	.0309	19.10		251.80	12.97	38.97
27-28	18.8	.0426	18.40		232.70	12.37	39.37
28-29	18.0	.0278	17.75		214.30	11.90	39.90

----

[1] 1 = age interval; 2 = percent surviving at beginning of age interval; 3 = retirement ratios; 4 = average ordinate for age interval; 5 = interval area; 6 = remaining area to the right of the ordinate at the beginning of the age interval; 7 = expectancy at beginning of age interval; 8 = probable life of average unit at beginning of age interval

Table 8. (Continued)

1'	2'	3'	4'	5'	6'	7'	8'
29-30	17.5	.0286	17.25		196.55	11.23	40.23
30-31	17.0	.0294	16.75		179.30	10.54	40.54
31-32	16.5	.0303	16.25		162.55	9.85	40.85
32-33	16.0	.0438	15.65		146.30	9.14	41.14
33-34	15.3	.0523	14.90		130.65	8.53	41.53
34-35	14.5	.0345	14.25		115.75	7.98	41.98
35-36	14.0	.0429	13.70		101.50	7.25	42.25
36-37	13.4	.0597	13.00		87.80	6.55	42.55
37-38	12.6	.0476	12.30		74.80	5.93	42.93
38-39	12.0	.0833	11.50		62.50	5.20	43.20
39-40	11.0	.0909	10.50		51.00	4.63	43.63
40-41	10.0	.1000	9.50		40.50	4.05	44.05
41-42	9.0	.1111	8.50		31.00	3.44	44.44
42-43	8.0	.1250	7.50		22.50	2.81	44.81
43-44	7.0	.2143	6.25		15.00	2.14	45.14
44-45	5.5	.2727	4.75		8.75	1.59	45.59
45-46	4.0	.5000	3.00		4.00	1.00	46.00
46-47	2.0	1.0000	1.00		1.00	.50	46.50
				1145.53			

$$\text{Average Service Life} = \frac{1145.53}{100} = 11.45 \text{ years}$$

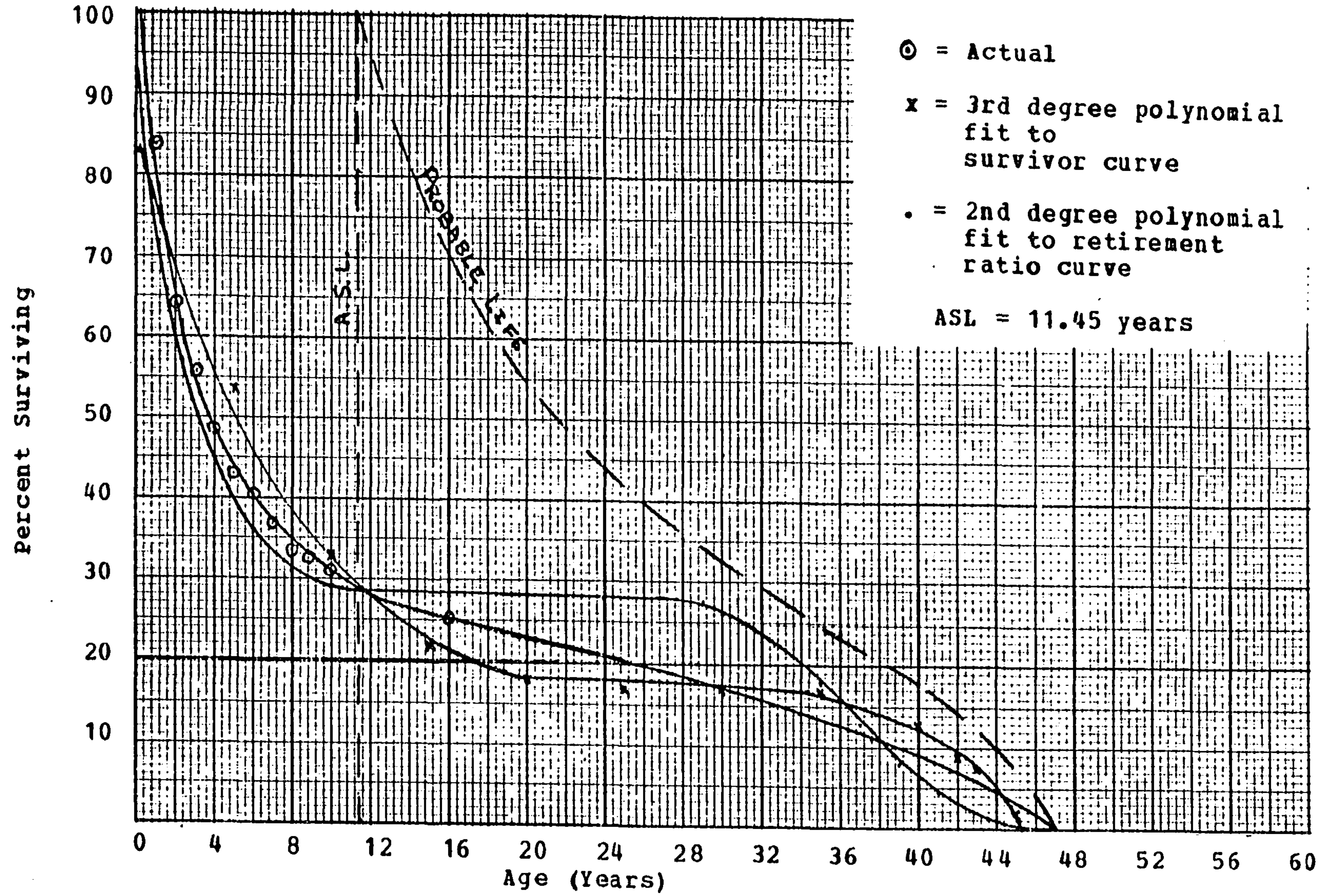

Figure 4. Polynomial fit to survivor curve and retirement ratio curve, Office Career College Graduates, 1961-71 original group

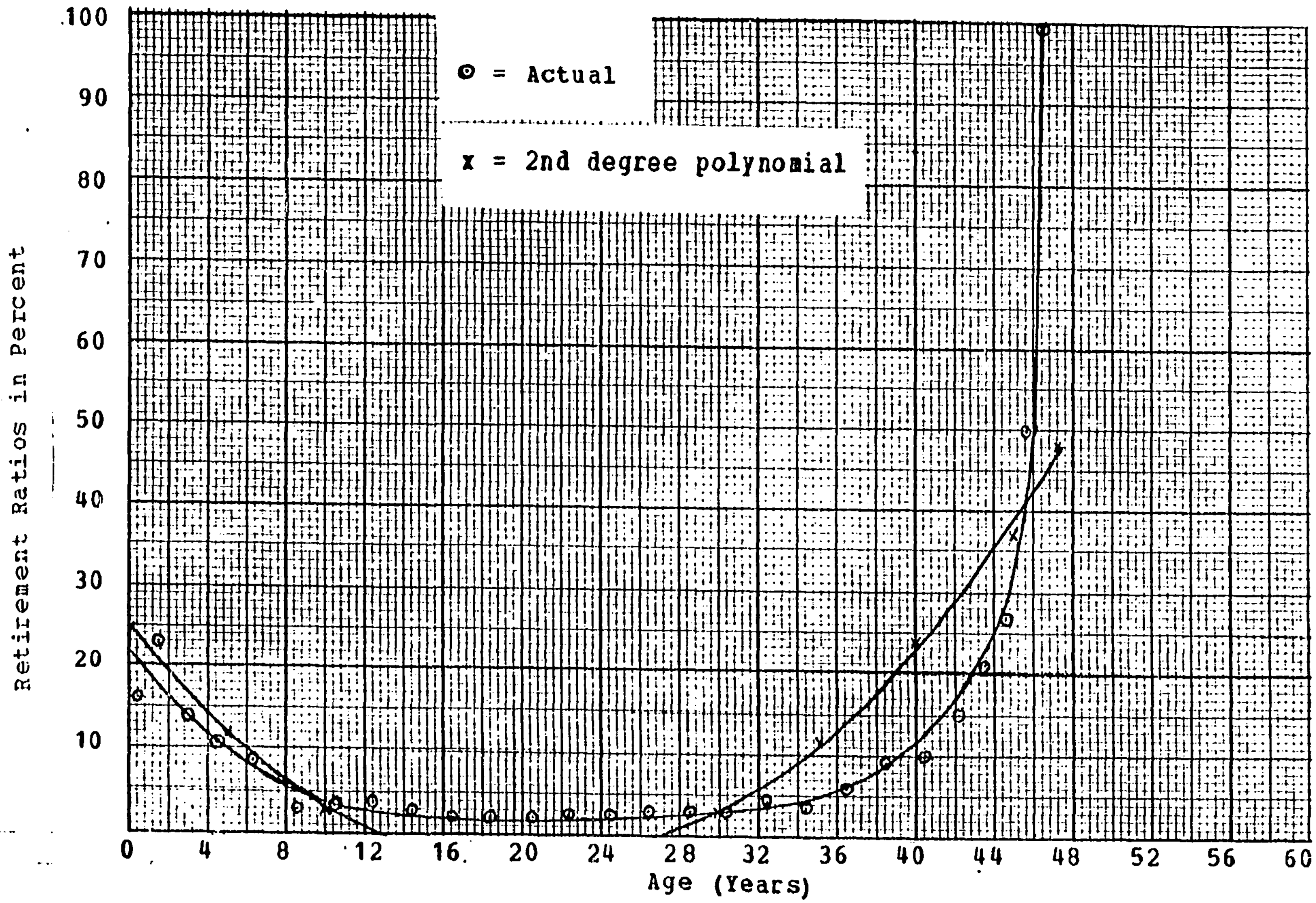

Figure 5. Retirement ratio curves, Office Career College Graduates, 1961-71 original group, actual and smoothed

APPLICATION TO INVESTMENT DECISIONS

The theory presented previously on engineering valuation, engineering economy and Markov chains is here applied to human resource valuation.

A program for computing present worth of future services

A computer program is presented here and in Appendix A (Figure 4) that will allow a manager to find the present worth of the compensation paid to employees in the department, assuming a constant annual percentage increase. The program applies the gradient theory presented earlier and uses the following notation:

Let PW = present worth of future compensation, p
C = present salary, c
R = annual salary increase in decimal percent, r
AINT = value of money in decimal percent, i
N = job life expectancy, n

Job life expectancy (N) is based on the survivor curves developed previously in this paper, or more appropriately for a particular company, (N) should take into consideration survivor curves drawn for the specific jobs.

To illustrate the use of this program, suppose a department manager in the company that contributed the Career College Graduate data had six college graduates in the department. The average service life for these people was 8 years and the retirements followed an Iowa O4 dispersion.

Knowing the present length of service for each individual, the Iowa 04 table can be used to find life expectancy since probable life is given in the table as a percent of average life and expectancy equals probable life minus age. To prepare the input to the program, a table like this was completed:

Dispersion Iowa 04

Average Service Life = 8 years

Time Value of Money = 8%

Department = Accounting

Name	Present Length of Service	Probable Life	Life Expectancy	Present Salary	Predicted Annual Salary Increase
V. H. Phillips	1	9.2	8.2	8000.00	.07
D. C. Jensen	10	20.5	10.5	16000.00	.05
J. L. Garcia	15	24.3	9.3	25000.00	.05
F. C. Deter	5	14.9	9.9	10000.00	.10
B. E. Weber	6	16.3	10.3	11000.00	.08
L. D. Uthe	2	10.5	8.5	9000.00	.12

Output from this program, Figure 7, shows the present worth of future earnings.

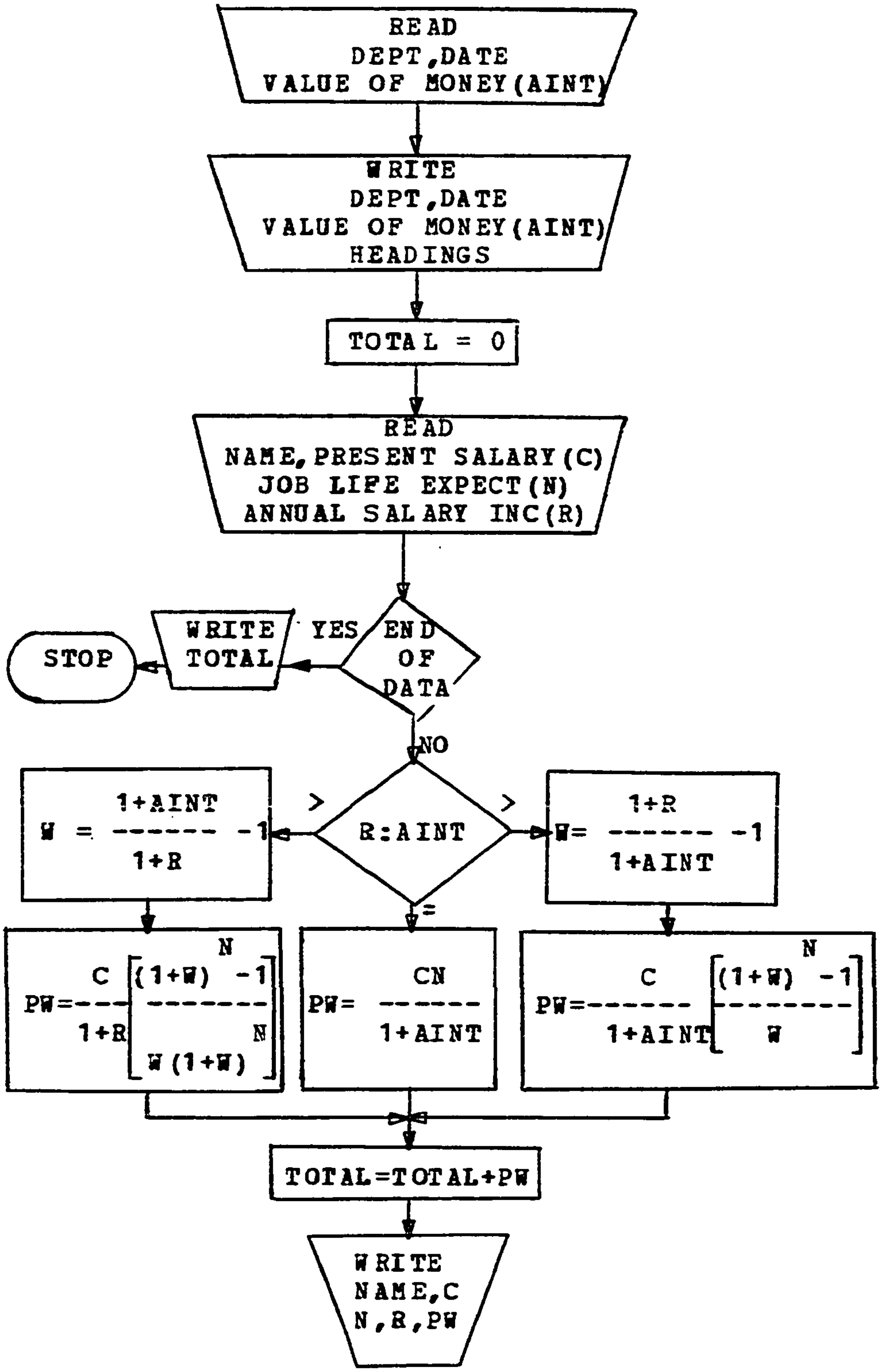

Figure 6. Flow chart for present worth of future services

DEPARTMENT: ACCOUNTING

DATE: OCTOBER 18, 1974

INTEREST RATE: 0.080

NAME	PRESENT SALARY	JOB LIFE EXPECTANCY	AVERAGE ANNUAL SALARY INCREASE IN DECIMAL %	PRESENT WORTH OF FUTURE EARNINGS IN $
V. H. PHILLIPS	8000.00	8.2	0.070	58754.55
D. C. JENSEN	16000.00	10.5	0.050	136564.50
J. L. GARCIA	25000.00	9.3	0.050	192066.88
F. C. DETER	10000.00	9.9	0.100	99600.50
B. E. WEBER	11000.00	10.3	0.080	104907.31
L. D. UTHE	9000.00	8.5	0.120	81502.31

				673396.00

Figure 7. Present worth of future services computer output

An economic justification

The impact of human resource accounting on managerial decisions is the subject of research (12), since better decision-making is one of the motivating reasons for determining the value of human resources.

To provide a basis for further discussion, a hypothetical situation is presented. Assume an investment is being considered with equipment costs of $200,000, 10 year life, zero salvage, straight line depreciation and annual operating costs of $20,000. Yearly revenue generated by the investment is $150,000. Four additional people are required, each having an annual salary of $15,000 with a $1,000 annual increase and 12 year job life expectancy. The constant gradient approach, g = $1,000, is used here.

$$\text{Present Worth of Future Salary} = 4\left[(15{,}000)\,(p/a)^{5\%}_{12} + 1{,}000\,(p/g)^{5\%}_{12}\right]$$

$$= 60{,}000(8.863) + 4{,}000(43.624)$$

$$= \$706{,}276$$

Next, the return on investment by both conventional and human resource methods could be examined.

	People as Operating Expense A	People as Capital Expenditure B
Revenue	150,000	150,000
Operating Expense 20,000 + 4 (15,000) + 4 (1,000) (4.922) =	99,688	20,000
Before Tax Cash Flow	50,312	130,000
Depreciation $\frac{200,000}{10}$	20,000	20,000
$\frac{706,276}{12}$		58,856
Taxable Income	30,312	51,144
Income Tax @ 50%	15,156	25,572
After Tax Cash Flow	35,156	104,428

Rate of Return Plan A

$$150,000 = 99,688 + 200,000 (a/p)^{i}_{10} + 15,156$$

$$(a/p)^{i}_{10} = \frac{35,156}{200,000} = .1758$$

$$i = 11.8\%$$

Rate of Return Plan B
(Assuming a return on both physical and human assets)

$$150{,}000 = 20{,}000 + 200{,}000(a/p)_{10}^{i} + 25{,}572 + 706{,}276(a/p)_{12}^{i}$$

$$104{,}428 = 200{,}000(a/p)_{10}^{i} + 706{,}276(a/p)_{12}^{i}$$

$$i = 5.6\%$$

If the minimum attractive rate of return required is 11%, but people are a scarce resource in this organization, it might be worthwhile to investigate other opportunities where the rate of return on the total investment, including human resources is greater than 5.6%.

Another alternative, and perhaps a more reasonable approach, would be to depreciate the human resources, but not deduct this from taxable income since the tax laws do not allow this deduction. The result for Plan B would then be:

Revenue	150,000
Operating Expense	20,000
Before Tax Cash Flow	130,000
Depreciation $\frac{706{,}276}{12} + \frac{200{,}000}{10} =$	78,856
Taxable Income(Same as Plan A)	30,312
Income Tax(Same as Plan A)	15,156

After Tax Cash Flow	114,844
Minus Dep. of Human Res.	58,856
Adjusted After Tax Cash Flow	55,988

Rate of Return Plan B

$$55,988 = 200,000(a/p)^{i}_{10} + 706,276(a/p)^{i}_{12}$$

$$i < 0$$

Woodruff (50), Pyle (40) and others would leave salaries as an expense and capitalize only the long range investments in human assets. Those costs capitalized include recruiting, acquisition costs such as moving and physical examinations, formal training, informal training and familiarization, and investment building after initial familiarization.

<u>Markov chain example</u>

Assume an employee starts in job 1 and expected service life = 3 years. There are a total of 3 jobs with the following transition matrix.

		To Job		
		1	2	3
From	1	.7	.2	.1
job	2	.1	.7	.2
	3	.1	.1	.8

The value expected to be derived from each job over a year's time is job 1 = $10,000, job 2 = $20,000, job 3 = $30,000.

$$E(S) = 10,000 + 10,000(p_{11}^{(1)}) + 20,000(p_{12}^{(1)}) + 30,000(p_{13}^{(1)}) + 10,000(p_{11}^{(2)}) + 20,000(p_{12}^{(2)}) + 30,000(p_{13}^{(3)})$$

In order to calculate $p_{ij}^{(2)}$, the two step transition probabilities are equal to $P \cdot P$.

$$\begin{bmatrix} .7 & .2 & .1 \\ .1 & .7 & .2 \\ .1 & .1 & .8 \end{bmatrix} \cdot \begin{bmatrix} .7 & .2 & .1 \\ .1 & .7 & .2 \\ .1 & .1 & .8 \end{bmatrix} = \begin{bmatrix} .52 & .29 & .19 \\ .16 & .53 & .31 \\ .16 & .17 & .67 \end{bmatrix} = P^{(2)}$$

Since the employee in this example began in job 1, only the first row of the 2-stage transition matrix is needed.

$$E(S) = 10,000 + 10,000(.7) + 20,000(.2) + 30,000(.1) + 10,000(.52) + 20,000(.29) + 30,000(.19) = \$40,700$$

The present worth of E(S) at 10 percent interest is then

$$10,000(p/f)_1^{10} + 10,000(.7)(p/f)_2^{10} + 20,000(.2)(p/f)_2^{10} + 30,000(.1)(p/f)_2^{10} + 10,000(.52)(p/f)_3^{10} + 20,000(.29)(p/f)_3^{10} +$$

$30{,}000(.19)(p/f)_3^{10} = \$33{,}208$

The steady-state equations can be expressed as

$$\begin{aligned}
\pi_1 &= \pi_1 p_{11} + \pi_2 p_{21} + \pi_3 p_{31} \\
\pi_2 &= \pi_1 p_{12} + \pi_2 p_{22} + \pi_3 p_{32} \\
\pi_3 &= \pi_1 p_{13} + \pi_2 p_{23} + \pi_3 p_{33} \\
1 &= \pi_1 + \pi_2 + \pi_3
\end{aligned}$$

Substituting values for p_{ij} into the last three equations since one is redundant

$$\begin{aligned}
\pi_2 &= .2\pi_1 + .7\pi_2 + .1\pi_3 \\
\pi_3 &= .1\pi_1 + .2\pi_2 + .8\pi_3 \\
1 &= \pi_1 + \pi_2 + \pi_3
\end{aligned}$$

Solving these equations provides the simultaneous solutions,

$$\begin{aligned}
\pi_1 &= .250 \\
\pi_2 &= .312 \\
\pi_3 &= .438
\end{aligned}$$

The n-state transition matrix when $n \rightarrow \infty$ is

$$P = \begin{bmatrix} .250 & .312 & .438 \\ .250 & .312 & .438 \\ .250 & .312 & .438 \end{bmatrix}$$

The 4-step transition probabilities are

$$P^{(4)} = \begin{bmatrix} .346 & .337 & .317 \\ .218 & .380 & .402 \\ .217 & .250 & .533 \end{bmatrix}$$

It can be seen that the 4-step probabilities are approaching the steady-state probabilities. In other words, after many years, assuming they stay, the probability of finding an employee in job 1, 2, or 3 is .250, .312, and .438 respectively.

If non-stationary transition probabilities are available from retirement ratio curves drawn for transition between jobs and exiting the company, then (a), (b) and (c) might represent these non-stationary probabilities and the n-step transition probabilities would be

(a)

$$P^{(1)} = \begin{bmatrix} .6 & .3 & 0 & .1 \\ .1 & .6 & .2 & .1 \\ .1 & .1 & .7 & .1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b)

$$P^{(2)} = \left[P^{(1)}\right] \cdot \begin{bmatrix} .4 & .3 & .1 & .2 \\ 0 & .5 & .3 & .2 \\ 0 & .2 & .6 & .2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} .24 & .33 & .15 & .28 \\ .04 & .37 & .31 & .28 \\ .04 & .22 & .46 & .28 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c)

$$P^{(3)} = \left[P^{(2)}\right] \cdot \begin{bmatrix} .2 & .4 & .2 & .2 \\ 0 & .4 & .3 & .3 \\ 0 & 0 & .5 & .5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} .048 & .228 & .222 & .502 \\ .008 & .164 & .274 & .554 \\ .008 & .104 & .304 & .584 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The expected value of future services in this case for an employee starting in job 1 is

$$\begin{aligned} E(S) &= 10{,}000 + 10{,}000(p_{11}^{(1)}) + 20{,}000(p_{12}^{(1)}) + 30{,}000(p_{13}^{(1)}) \\ &\quad + 10{,}000(p_{11}^{(2)}) + 20{,}000(p_{12}^{(2)}) + 30{,}000(p_{13}^{(2)}) \\ &\quad + 10{,}000(p_{11}^{(3)}) + 30{,}000(p_{12}^{(3)}) + 30{,}000(p_{13}^{(3)}) + \ldots\ldots \\ &= 10{,}000 + 10{,}000(.6) + 20{,}000(.3) + 30{,}000(0) \\ &\quad + 10{,}000(.24) + 20{,}000(.33) + 30{,}000(.15) \\ &\quad + 10{,}000(.048) + 20{,}000(.228) + 30{,}000(.222) + \ldots\ldots \end{aligned}$$

This is continued until the present worth of an additional period would add an insignificant amount to the total value of future services or until all values in the n-stage matrix are zero except for the exit probabilities. Computer programs are available to perform these multiplications easily, once the transition probabilities have been established from the retirement ratio curves.

CONCLUSIONS

Primary

1. Life analysis techniques should prove extremely useful in the valuation of human resources. The survivor, retirement frequency, and retirement ratio curves, along with the techniques for their determination, are as applicable in human resource valuation as they are in industrial property valuation.

2. The Iowa survivor curves will adequately describe service life characteristics if the average service lives are of sufficient length. It is difficult to prescribe specific guidelines. The Iowa curves did not fit the Utility company personnel when the majority of the employees left the first year, but many of those who remained stayed for over 20 years. This resulted in a survivor curve with an initial sharp drop and a long, horizontal tail. A rough guideline might be to examine the employee separation characteristics the first year. If there is a reasonable loss of 10 to 50 percent, the Iowa curves will fit, whereas a 70 percent or greater loss cannot be described by the Iowa curves.

3. When the Iowa curves did fit the survivor characteristics, in almost all cases it was an O type curve, usually an O4. This curve bears a resemblance to a negative exponential and

was developed after the original Iowa curves. The two exceptions came from the simulated plant balances summary which indicated that a R1 and L2 curve was appropriate.

Secondary

1. Survivor curve shape characteristics have been investigated adequately elsewhere (20), but a cursory examination of the curves obtained in this research indicated two things. First, even though a negative exponential would seem to be a logical choice, it will not describe this data. However, the overall resemblance indicates that it might be used for estimation purposes. Second, the method of fitting a polynomial to retirement ratios and converting to a survivor curve is still appropriate.

2. The average service lives for the utility, office, and manufacturing organizations are shown in Tables 4, 5 and 6. The length of service for the utility employees was on the order of 3 to 5 years, while the other two companies went from 3 to 18. Laborers, Ironworkers, and Clerical employees had very short service lives of .5 to 3 years, while in contrast, the Marketing employees went up to 18 years in the case of one original group. These lives could be guidelines for other organizations in the event they were to investigate a human resource measurement program. However, a more

desirable approach would be to use the techniques and programs presented here to establish survivor curves tailored to the particular organization.

Further research possibilities include refining the average service life characteristics, since it would be desirable to obtain data showing the length of time employees spend in particular jobs within the same company. This would add accuracy to calculations of resource value regardless of whether future salary or future contribution is used as an indicator, and enable the establishment of characteristic non-stationary probability transition matrices for the Markov chain approach. Further study of actual retirement data might show the need for additional standard survivor curves that more accurately describe human service life characteristics. The depreciation rate of human assets, the effect of human resource value on investment decisions, and a method for establishing the contribution of the jobs of an organization to the total services provided by the organization could all be further investigated.

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ACKNOWLEDGEMENTS

I would like to thank Dr. Harold A. Cowles for his overall guidance and the direction given in analogies between the valuation of physical property and employees. Also, Dr. Richard W. Mensing was very helpful in the statistical comparison between the Iowa survivor curves and the experimental data. The three companies providing data, although they would prefer not to be mentioned by name, were especially generous with their time and resources. Without their cooperation, this research could not have been started. Financial support was provided through the efforts of Dr. Clifford E. Smith and the American Society for Personnel Administration. This support was truly appreciated and I hope any contribution made to human resource valuation resulting from this research will compensate A.S.P.A. for their generosity.

Sheryl, Chris, and Laura were very patient and understanding in allowing me the long hours necessary to complete this project.

APPENDIX A: COMPUTER PROGRAM LISTINGS

```
FORTRAN IV G LEVEL  21                    MAIN

          C     THIS PROGRAM COMPUTES THE PRESENT WORTH OF FUTURE SERVICES,
          C     ASSUMING THAT SALARY IS A GOOD ESTIMATE OF THE WORTH OF THOSE
          C     SERVICES.
          C     THE FIRST INPUT CARD CONTAINS THE DEPARTMENT NAME IN COLUMES
          C     1-40, DATE 41-60, VALUE OF MONEY(AINT) 61-65.
          C     EACH SUCCEEDING CARD CONTAINS NAME IN COLUMES 1-24, PRESENT
          C     SALARY(C) 25-33, JOB LIFE EXPECTANCY(EX) 34-38,
          C     ANNUAL INCREASE(R) 39-43.
          C
                DIMENSION DEPT(10),DATE(5),NAME(6)
                INTEGER*4 DEPT,DATE,NAME
                READ(5,2)DEPT,DATE,AINT
              2 FORMAT(10A4,5A4,F5.3)
                WRITE(6,4)DEPT,DATE,AINT
              4 FORMAT('1   DEPARTMENT: ',10A4//,'          DATE: ',5A4//,
               -' INTEREST RATE: ',F5.3)
                WRITE(6,6)
              6 FORMAT(1H048X,'AVERAGE ANNUAL     PRESENT WORTH'/' ',26X,'PRES
               -  JOB LIFE    SALARY INCREASE    OF FUTURE'/' ',9X,'NAME',13X,
               -'SALARY   EXPECTANCY     IN DECIMAL %     EARNINGS IN $',/)
                TOTAL=0.0
              7 READ(5,8,END=50)NAME,C,EX,R
              8 FORMAT(6A4,F9.2,F5.1,F5.3)
```

Figure 8. Present worth of future services

```
C      C=PRESENT SALARY, R=ANNUAL INCREASE IN DECIMAL %, EX=JOB LIFE
C      EXPECTANCY, AINT=VALUE OF MONEY IN DECIMAL %
       IF(R.GT.AINT) GO TO 10
       IF(R.LT.AINT) GO TO 12
       PW=(C*EX)/(1.0+AINT)
       GO TO 14
   10  W=(1.0+R)/(1.0+AINT)-1.0
       PW=(C/(1.0+AINT))*(((1.0+W)**EX-1.0)/W)
       GO TO 14
   12  W=(1.0+AINT)/(1.0+R)-1.0
       PW=(C/(1.0+R))*(((1.0+W)**EX-1.0)/(W*(1.0+W)**EX))
   14  TOTAL=TOTAL+PW
       WRITE(6,16)NAME,C,EX,R,PW
   16  FORMAT(1X6A4,1X,F9.2,4X,F5.1,10X,F5.3,11X,F10.2)
       GO TO 7
   50  WRITE(6,18)
   18  FORMAT(70X,'----------')
       WRITE(6,20)TOTAL
   20  FORMAT(68X,F12.2)
       STOP
       END
```

Figure 8. (Continued)

```
$JOB      'BARTA',TIME=5,PAGES=10
C      THIS PROGRAM CALCULATES A CHI-SQUARED STATISTIC  BASED ON THE
C      DIFFERENCE BETWEEN ACTUAL RETIREMENTS AND RETIREMENTS TAKEN FROM
C      THE BEST FITTING IOWA CURVE.
C      THE PROBABILITIES OF RETIREMENT COME FROM INTEGRATION OF THE
C      RETIREMENT FREQUENCY CURVE, ANOTHER COMPUTER PROGRAM.
C      T.A.BARTA, I.S.U., JUNE 1, 1974
C      RETIREMENT RATE CHI-SQUARE PROGRAM
       REAL PROB,ACCUM,ERET
       INTEGER ARET,I,N
       DIMENSION ARET(30),PROB(30),ERET(30),NEXP(30)
       READ(5,10)N
C      N=THE NUMBER OF CLASSES, ARRANGED SO THERE ARE AT LEAST 5
C        RETIREMENTS PER CLASS IF POSSIBLE.
    10 FORMAT(I2)
       READ(5,15)(ARET(I),I=1,N)
    15 FORMAT(16I5)
C      ARET(I) CONTAINS THE ACTUAL RETIREMENTS IN EACH CLASS
       READ(5,20)(NEXP(I),I=1,N)
    20 FORMAT(16I5)
C      NEXP= THE # OF PEOPLE IN THE ORIGINAL GROUP
       READ(5,25)(PROB(I),I=1,N)
    25 FORMAT(8F10.7)
C      PROB(I) CONTAINS THE PROB. OF RETIRING DURING EACH INTERVAL
C        IN DECIMAL FORM.
       ACCUM=0
       DO 30 I=1,N
       ERET(I)=NEXP(I)*PROB(I)
C      ERET(I)= EXPECTED # OF RETIREMENTS
```

Figure 9. Retirement rate chi-square test

```
      IF(NEXP(I).EQ.0)GO TO 30
      ACCUM=ACCUM+((ARET(I)-NEXP(I)*PROB(I))**2)/(NEXP(I)*PROB(I)*(1-PR
     1OB(I))))
      WRITE(6,27)ACCUM
   27 FORMAT(' ','ACCUM = ',F12.6)
   30 CONTINUE
      WRITE(6,32)ACCUM
   32 FORMAT('0','CHI-SQUARE STATISTIC= ',F12.6)
      WRITE(6,35)(ARET(I),I=1,N)
   35 FORMAT(' ','ACTUAL RETIREMENTS= ',13I5)
      WRITE(6,37)(ERET(I),I=1,N)
   37 FORMAT(' ','EXPECTED RETIREMENTS= ',13F5.1)
      WRITE(6,40)(NEXP(I),I=1,N)
   40 FORMAT(' ','ORIGINAL GROUPS = ',13I5)
      WRITE(6,45)(PROB(I),I=1,N)
   45 FORMAT(' ','PROB. OF RETIRE.= ',10F10.7)
      STOP
      END
```

```
CHI-SQUARE STATISTIC=       8.090201
ACTUAL RETIREMENTS=      7    6    7    4    3    1    1    1

EXPECTED RETIREMENTS=   7.9  4.6  4.1  2.4  1.6  0.7  0.6  0.3
ORIGINAL GROUPS =     38   26   31   27   25   15   16   10
PROB. OF RETIRE.=  0.2088721 0.1772131 0.1312222 0.0902599 0.0624076 0.0451168
```

Figure 9. (Continued)

```
$JOB      'BARTA',TIME=5,PAGES=10
      C     THIS PROGRAM CALCULATES A CHI-SQUARED STATISTIC  BASED ON THE
      C     DIFFERENCE BETWEEN ACTUAL RETIREMENTS AND RETIREMENTS TAKEN FROM
      C     THE BEST FITTING IOWA CURVE.
      C     THE PROBABILITIES OF RETIREMENT COME FROM INTEGRATION OF THE
      C     RETIREMENT FREQUENCY CLRVE, ANOTHER COMPUTER PROGRAM.
      C     T.A.BARTA, I.S.U., MAY 30, 1974
      C     ORIGINAL GROUP CHI-SQUARE PROGRAM
            REAL PROB,ACCUM,ERET
            INTEGER ARET,I,N
            DIMENSICN ARET(30),PROB(30),ERET(30)
            READ(5,10)N
      C     N=THE NUMBER OF CLASSES, ARRANGED SO THERE ARE AT LEAST 5
      C       RETIREMENTS PER CLASS IF POSSIBLE.
         10 FORMAT(I2)
            READ(5,15)(ARET(I),I=1,N)
         15 FORMAT(16I5)
      C     ARET(I) CONTAINS THE ACTUAL RETIREMENTS IN EACH CLASS
            READ(5,20)NEXP
         20 FORMAT(I5)
      C     NEXP= THE # OF PEOPLE IN THE ORIGINAL GROUP
            READ(5,25)(PROB(I),I=1,N)
         25 FORMAT(8F10.7)
      C     PROB(I) CONTAINS THE PROB. OF RETIRING DURING EACH INTERVAL
      C       IN DECIMAL FORM.
            ACCUM=0
            CO 30 I=1,N
            ERET(I)=NEXP*PROB(I)
            ACCUM=ACCUM+(ARET(I)-NEXP*PROB(I))**2/(NEXP*PROB(I))
```

Figure 10. Original group chi-square test

```
   30 CONTINUE
      WRITE(6,32)ACCUM
   32 FORMAT('0','CHI-SQUARE STATISTIC= ',F12.6)
      WRITE(6,35)(ARET(I),I=1,N)
   35 FORMAT(' ','ACTUAL RETIREMENTS= ',13I5)
      WRITE(6,37)(ERET(I),I=1,N)
   37 FORMAT(' ','EXPECTED RETIREMENTS= ',13F5.1)
      WRITE(6,40)NEXP
   40 FORMAT(' ','ORIGINAL GROUP= ',I5)
      WRITE(6,45)(PROB(I),I=1,N)
   45 FORMAT(' ','PROB. OF RETIRE.= ',10F10.7)
      STOP
      END

$ENTRY
```

```
CHI-SQUARE STATISTIC=     14.330400
ACTUAL RETIREMENTS=      2    4    9    7    1    4    0    2    2    3   20
EXPECTED RETIREMENTS=  5.0  4.8  4.5  4.1  3.7  3.2  2.7  2.3  2.0  1.7 20.0
ORIGINAL GROUP=     54
PROB. OF RETIRE.=  0.0929183 0.0894839 0.0839597 0.0767005 0.0682129 0.0591213
PROB. OF RETIRE.=  0.0505048 0.0428612 0.0362560 0.030834  0.3697067
```

Figure 10. (Continued)

```
$JOB        'BARTA',TIME=5,PAGES=10
C       THIS PROGRAM PERFORMS INTEGRATION OF A GIVEN FUNCTION BY THE
C       TRAPEZOICAL RULE AND CAN BE FOUND ON PAGE 297 OF THE IBM
C       SCIENTIFIC SUBROUTINE PACKAGE. IT COMPUTES AN APPROXIMATION
C       FOR INTEGRAL FCT(X) SUMMED OVER X FROM XL TO XU.
C       FUNCTICN FCT SHOULD BE CHANGED DEPENDING ON THE RETIREMENT
C          FREQUENCY CURVE.
C         XL=LOWER BOUND OF INTERVAL.
C         XU=UPPER BOUND OF INTERVAL.
C        EPS=UPPER BOUND OF ABSOLUTE ERROR.
C       NDIM=DIMENSION OF THE AUXILIARY STORAGE ARRAY AUX.NDIM-1 IS THE
C              MAXIMUM # OF BISECTIONS OF THE INTERVAL (XL,XU).
C        FCT=THE NAME OF THE EXTERNAL FUNCTION SUBPROGRAM USED.
C          Y=THE RESULTING APPROXIMATION FOR THE INTEGRAL VALUES.
C        IER=A RESULTING ERROR PARAMETER.
C        AUX=AN AUXILIARY STORAGE ARRAY WITH DIMENSION NDIM.
C        ERROR PARAMETER (IER) CODE:
C           0=NO ERROR.IT WAS POSSIBLE TO REACH THE DESIRED ACCURACY.
C           1=IMPOSSIBLE TO REACH THE DESIRED ACCURACY DUE TO ROUNDING
C              ERRORS.
C           2=IMPOSSIBLE TO CHECK ACCURACY BECAUSE NDIM<5,OR THE REQUIRED
C              ACCURACY COULD NOT BE REACHED WITHIN NDIM-1 STEPS.NDIM
C              SHOULD BE INCREASED.
C       WRITTEN BY TOM BARTA, I.S.U., MAY 5,1974
C
1       EXTERNAL FCT
2       REAL XL,XU,EPS,Y,X,TOTAL
```

Figure 11. Integration of retirement ratio curve

```
      INTEGER NDIM,IER
      DIMENSION AUX(50)
      TOTAL=0
    5 READ(5,10,END=25)XL,XU,EPS,NDIM
   10 FORMAT(F7.3,F7.3,F4.2,I3)
      WRITE(6,15)XL,XU,EPS,NDIM
   15 FORMAT('0','XL=',F7.3,'  XU=',F7.3,'  EPS=',F4.2,'  NDIM=',I3)
      CALL CATR(XL,XU,EPS,NDIM,FCT,Y,IER,AUX)
      WRITE(6,20)IER,Y
   20 FORMAT(' ','ERROR PARAMETER= ',I1,/' ','INTEGRAL APPROXIMATION= ',
     1F12.5)
      TOTAL=TOTAL+Y
      GO TO 5
   25 WRITE(6,30)TOTAL
   30 FORMAT('0','TOTAL AREA= ',F12.6)
      STOP
      END
C
C
C     THIS FUNCTION SUBPROGRAM CONTAINS THE EQUATION TO BE INTEGRATED

      FUNCTICN FCT(X)
      FCT=11*(1+(.1*X+1.6)**6)**(-.6)+.59863384
      RETURN
      END

      $ENTRY

XL=-10.0CO  XU= -7.500  EPS=0.10  NDIM= 20
ERROR PARAMETER= 0
INTEGRAL APPROXIMATICN=     26.63618
```

Figure 11. (Continued)

```
XL= -7.5C0   XU= -5.000   EPS=0.10   NDIM= 20
ERROR PARAMETER= 0
INTEGRAL APPROXIMATION=      20.40346

XL= -5.00C   XU= -2.500   EPS=0.10   NDIM= 20
ERROR PARAMETER= 0
INTEGRAL APPROXIMATICN=      12.96222

XL= -2.5CC   XU=  0.000   EPS=0.10   NDIM= 20
ERROR PARAMETER= 0
INTEGRAL APPRCXIMATICN=       8.01120

XL=  0.000   XU=  2.500   EPS=0.10   NDIM= 20
ERROR PARAMETER= 0
INTEGRAL APPROXIMATICN=       5.32291

XL=  2.500   XU=  5.000   EPS=0.10   NDIM= 20
ERROR PARAMETER= 0
INTEGRAL APPROXIMATICN=       3.87068

XL=  5.000   XU= 34.000   EPS=0.10   NDIM= 20
ERROR PARAMETER= 0
INTEGRAL APPROXIMATICN=      22.84993

TOTAL AREA=   100.056500
```

Figure 11. (Continued)

```
$JOB      'BARTA',TIME=5,PAGES=10
C      THIS PROGRAM PERFORMS LEAST SQUARES POLYNOMIAL CURVE FITS FROM
C      THE FIRST TO THE TENTH DEGREE, WITH THE DEGREES SPECIFIED BY THE
C      FIRST DATA CARD.
C      THE FIRST DATA CARD CONTAINS THE LOWEST DEGREE IN COLUMES 1-2,
C      AND THE HIGHEST DEGREE IN COLUMES 3-4.
C      SUCCESSIVE DATA CARDS CONTAIN ONE X AND ONE Y VALUE PER CARD
C      IN F10.3 FORMAT. X IN COLUMES 1-10, Y IN COLUMES 11-20.
C      THE PROGRAM STOPS WHEN 9999.000 IS ENCOUNTERED IN THE FIRST 10
C      COLUMES OF A CARD.
C      THE OUTPUT GIVES THE VALUE OF THE CONSTANT FIRST, THEN X, THEN
C      X SQUARED ETC.
       DIMENSION X(200),Y(200),A(11,11),B(11),C(11),P(20)
       READ 20,M,IDEG
    20 FORMAT(2I2)
       DO 11 I=1,201
       READ 10,X(I),Y(I)
    10 FORMAT(2F10.3)
       IF(X(I).EQ.9999.000)GO TO 12
    11 CONTINUE
       STOP
    12 NUMBER=I-1
   100 MX2=M*2
       DO 13 I=1,MX2
       P(I)=0.0
       DO 13 J=1,NUMBER
    13 P(I)=P(I)+X(J)**I
       N=M+1
       DO 30 I=1,N
       DO 30 J=1,N
```

Figure 12. Least squares polynomial curve fit

```
      K=I+J-2
      IF(K)29,29,28
   28 A(I,J)=P(K)
      GO TO 30
   29 A(1,1)=NUMBER
   30 CONTINUE
      B(1)=0.0
      DO 21 J=1,NUMBER
   21 B(1)=B(1)+Y(J)
      DO 22 I=2,N
      B(I)=0.0
      DO 22 J=1,NUMBER
   22 B(I)=B(I)+Y(J)*X(J)**(I-1)
      NM1=N-1
      DO 300 K=1,NM1
      KP1=K+1
      L=K
      DO 400 I=KP1,N
      IF(ABS(A(I,K))-ABS(A(L,K)))400,400,401
  401 L=I
  400 CONTINUE
      IF(L-K)500,500,405
  405 DO 410 J=K,N
      TEMP=A(K,J)
      A(K,J)=A(L,J)
  410 A(L,J)=TEMP
      TEMP=B(K)
      B(K)=B(L)
      B(L)=TEMP
  500 DO 300 I=KP1,N
```

Figure 12. (Continued)

```
      FACTOR=A(I,K)/A(K,K)
      A(I,K)=0.0
      DO 301 J=KP1,N
  301 A(I,J)=A(I,J)-FACTOR*A(K,J)
  300 B(I)=B(I)-FACTOR*B(K)
      C(N)=B(N)/A(N,N)
      I=NM1
  710 IP1=I+1
      SUM=0.0
      DO 700 J=IP1,N
  700 SUM=SUM+A(I,J)*C(J)
      C(I)=(B(I)-SUM)/A(I,I)
      I=I-1
      IF(I)800,800,710
  800 PRINT 799,M
  799 FORMAT('0THE DEGREE',I3,' POLYNOMIAL FOLLOWS')
      DO 900 I=1,N
  900 PRINT 901,I,C(I)
  901 FORMAT(I5,F15.7)
      M=M+1
      IF(M-IDEG)100,100,902
  902 STOP
      END

$ENTRY

THE DEGREE  1 POLYNCMIAL FOLLOWS
   1      0.5778247
   2     -0.0130621
```

Figure 12. (Continued)

```
THE DEGREE  2 POLYNOMIAL FOLLOWS
   1      0.7089751
   2     -0.0325546
   3      0.0004133

THE DEGREE  3 POLYNOMIAL FOLLOWS
   1      0.8369154
   2     -0.0747247
   3      0.0027579
   4     -0.0000333

THE DEGREE  4 POLYNOMIAL FOLLOWS
   1      0.9060558
   2     -0.1141264
   3      0.0067803
   4     -0.0001688
   5      0.0000014

THE DEGREE  5 POLYNOMIAL FOLLOWS
   1      0.9125715
   2     -0.1216782
   3      0.0081980
   4     -0.0002595
   5      0.0000038
   6     -0.0000000
```

Figure 12. (Continued)

```
   5.           /*THIS PROGRAM WILL COMPUTE Y VALUES, GIVEN A
POLYNOMIAL EXPRESSION.  STATEMENT 50 CONTAINS THE EXPRESSION AND
BE CHANGED FOR EACH  DIFFERENT POLYNOMIAL EVALUATED.*/;
  10.                GET LIST(N);
  20.                I=0;
  50.      START:    Y=73.32345-7.21381*I+.271389*I*I-.0032341*I*I*I;
  60.                PUT LIST(Y);
  62.                I=I+1;
  65.                IF I>N THEN GO TO DONE;
  70.                GO TO START;
  80.      DONE:     END POLY;
?
```

Figure 13. Percent surviving, given a polynomial survivor curve

```
          /*GIVEN A POLYNOMIAL EXPRESSION FOR A RETIREMENT
RATIO CURVE, THIS PROGRAM WILL COMPUTE THE PERCENT SURVIVING FOR AGES
FROM 0 TO N IN INCREMENTS OF 1.*/;
          /*IF THE RETIREMENT RATIO GOES NEGATIVE, IT IS
ADJUSTED TO 0.*/;
          DECLARE  Y CHAR(100) VAR;
          PUT LIST('FOR HOW MANY YEARS DO YOU WANT TO EXTEND
THE SURVIVOR CURVE?');
          GET LIST(N);
          PUT LISJ('INPUT THE POLYNOMIAL EXPRESSION FOR Y. DO
NOT INCLUDE THE DEPENDENT VARIABLE Y OR THE = SIGN. USE THE LETTER I
FOR THE VARIABLE NAME.');
          READ INTO(Y) ;
          I=0;
          SURV=100;
  START:  PUT IMAGE(I,SURV)(IM);
  IM:     IMAGE;
  ---     ---.----
          I=I+1;
          IF I>N THEN GO TO DONE;
          RR=EVAL(Y);
          IF RR<0 THEN RR=0;
          SURNEW=SURV-RR*SURV;
          SURV=SURNEW;
          GO TO START;
  DONE:   END SURV;
?
```

Figure 14. Percent surviving, given a polynomial retirement ratio curve

APPENDIX B: COMPARISON OF STUB, SMOOTHED AND IOWA SURVIVOR CURVES

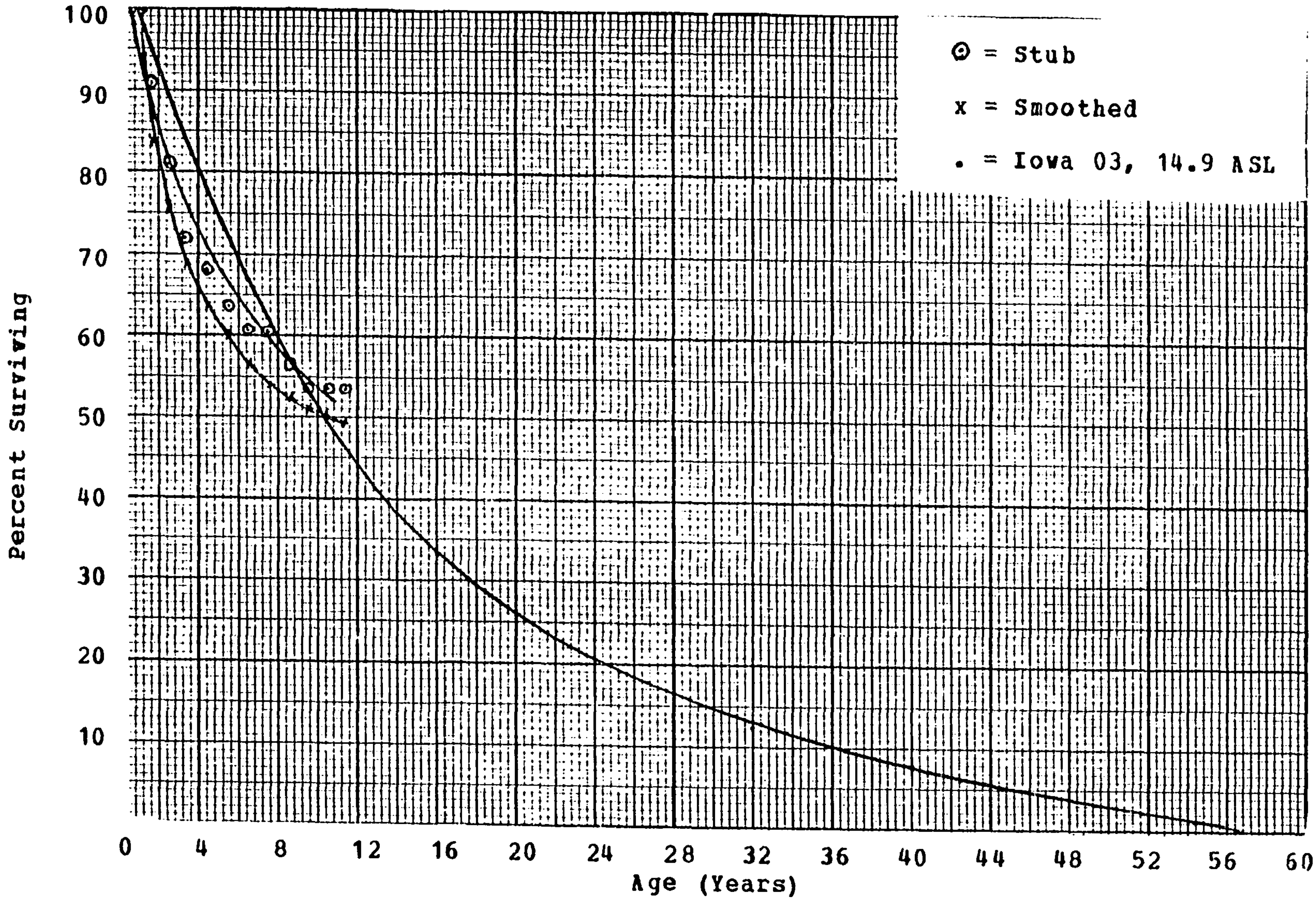

Figure 15. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Manufacturing Marketing, 1955-57 experience band

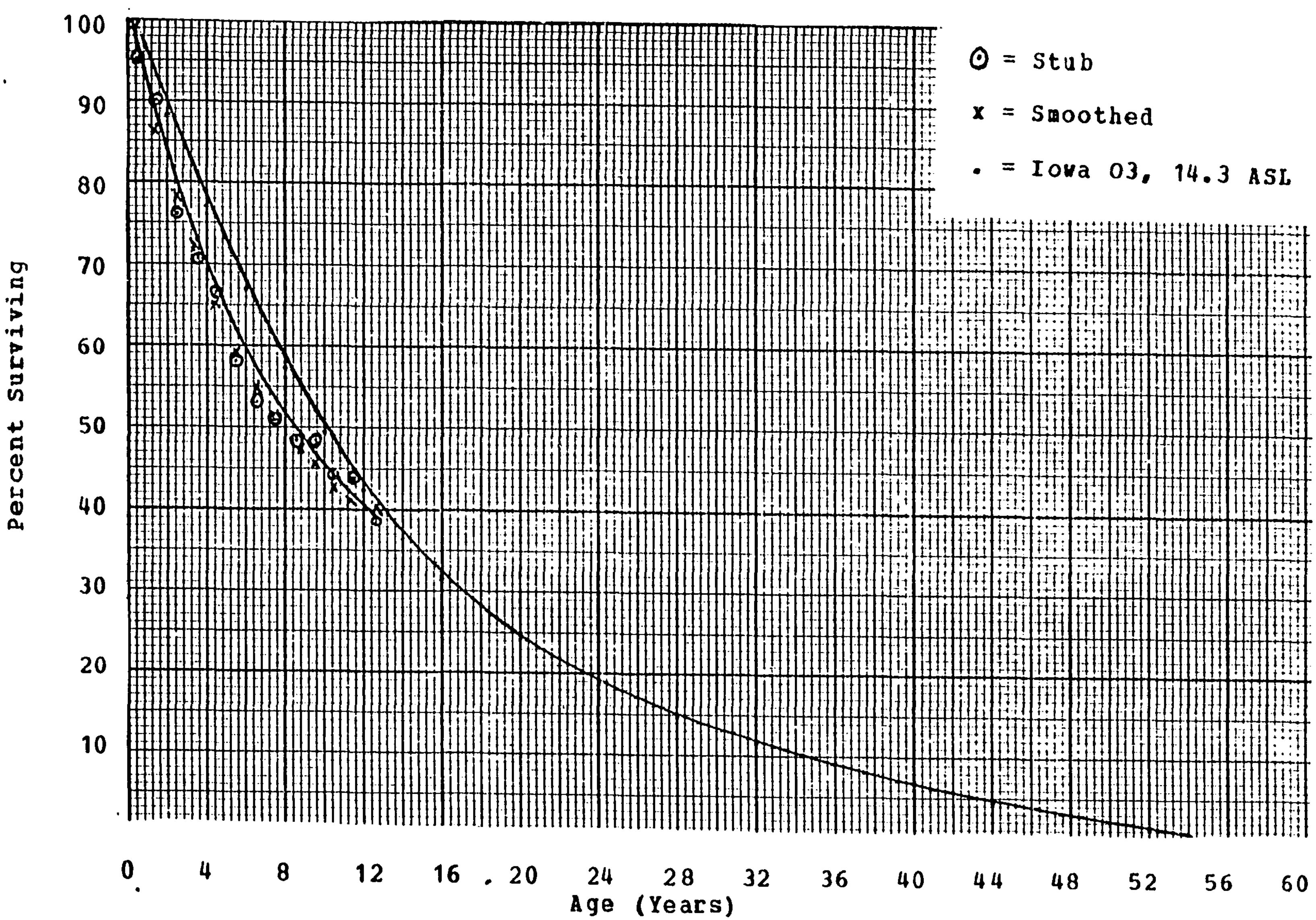

Figure 16. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Manufacturing Marketing, 1961-63 experience band

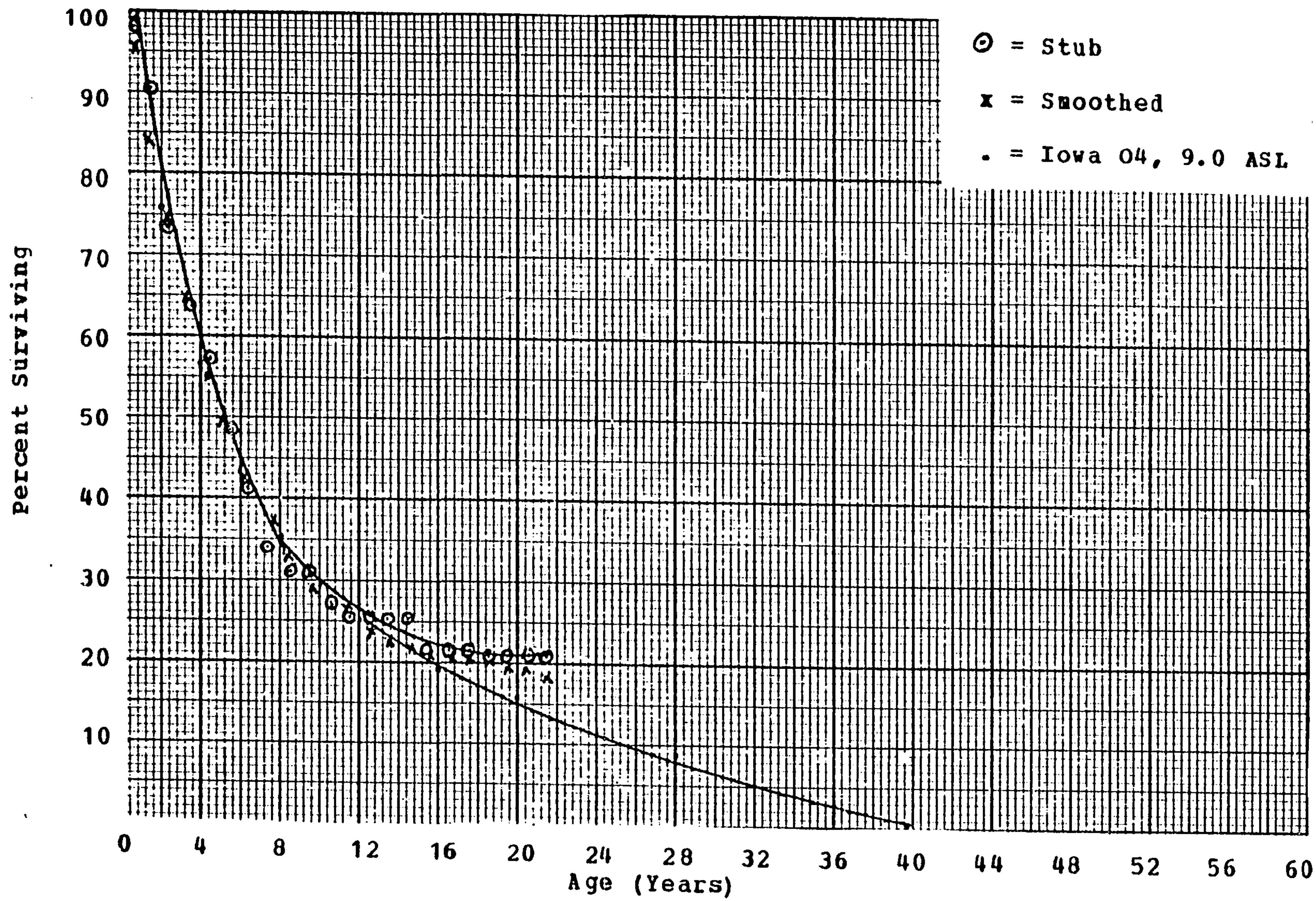

Figure 17. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Manufacturing Marketing, 1964-66 experience band

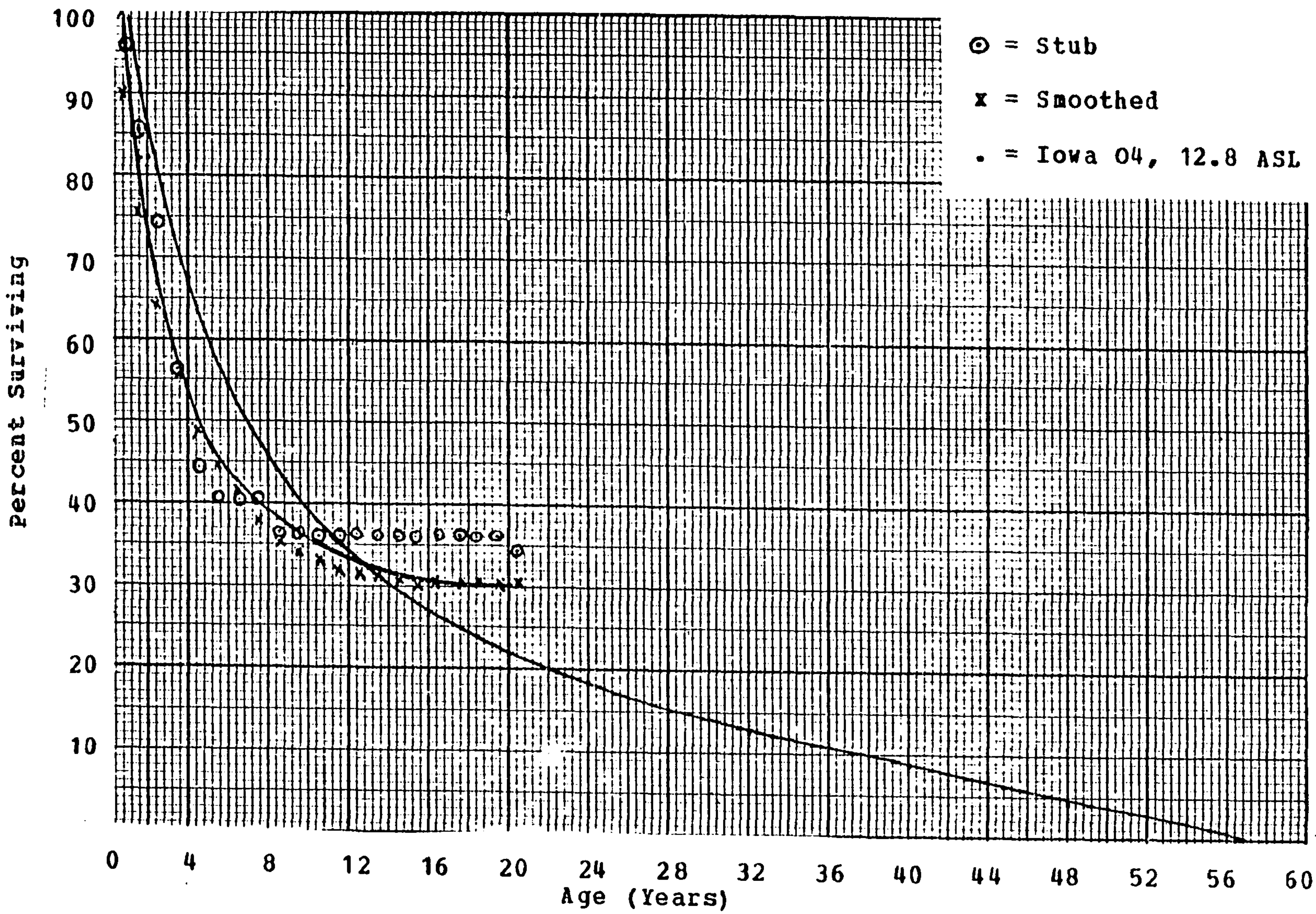

Figure 18. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Manufacturing Marketing, 1967-69 experience band

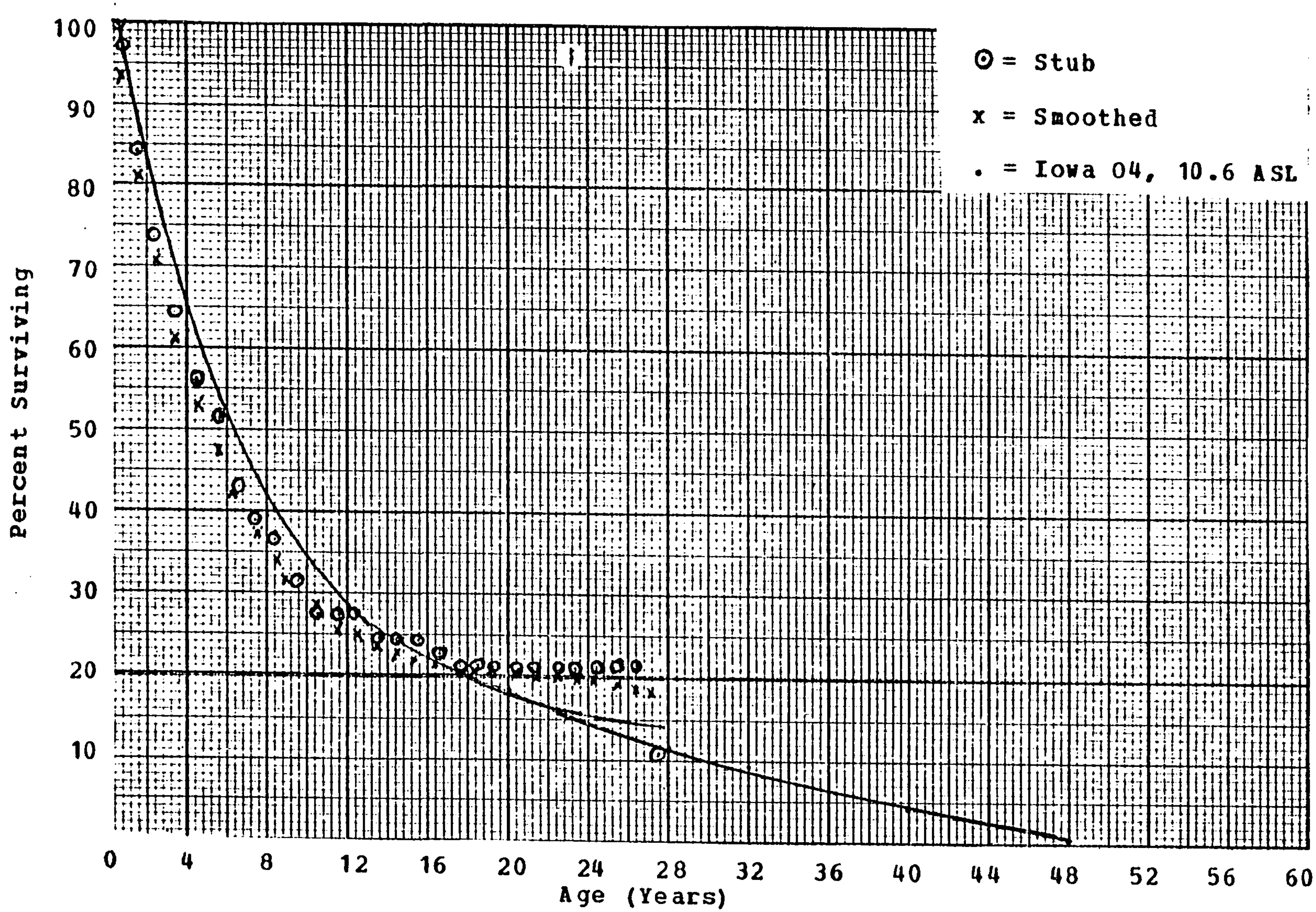

Figure 19. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Manufacturing Marketing, 1970-72 experience band

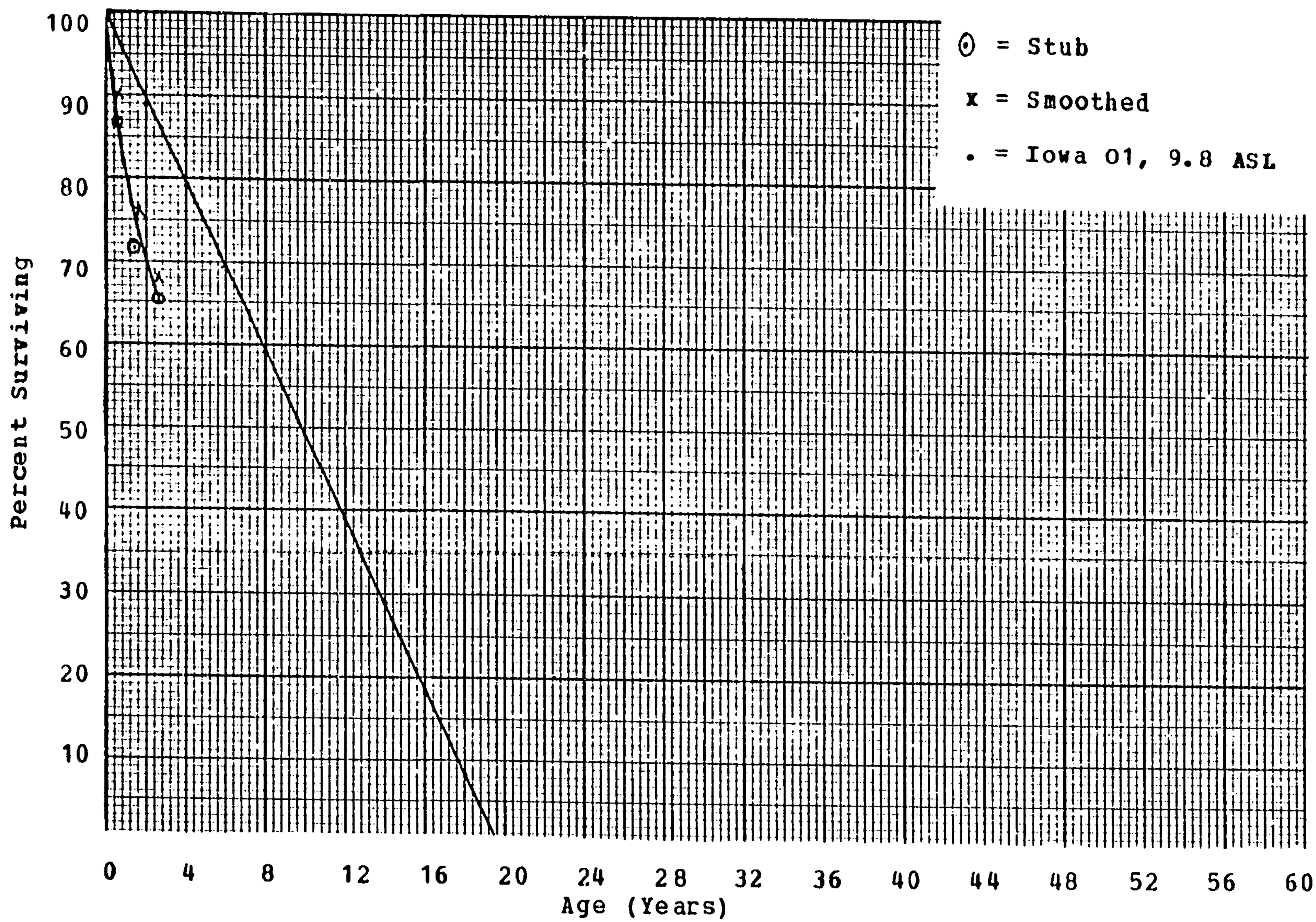

Figure 20. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Office, Career Career College Graduates, 1961-63 experience band

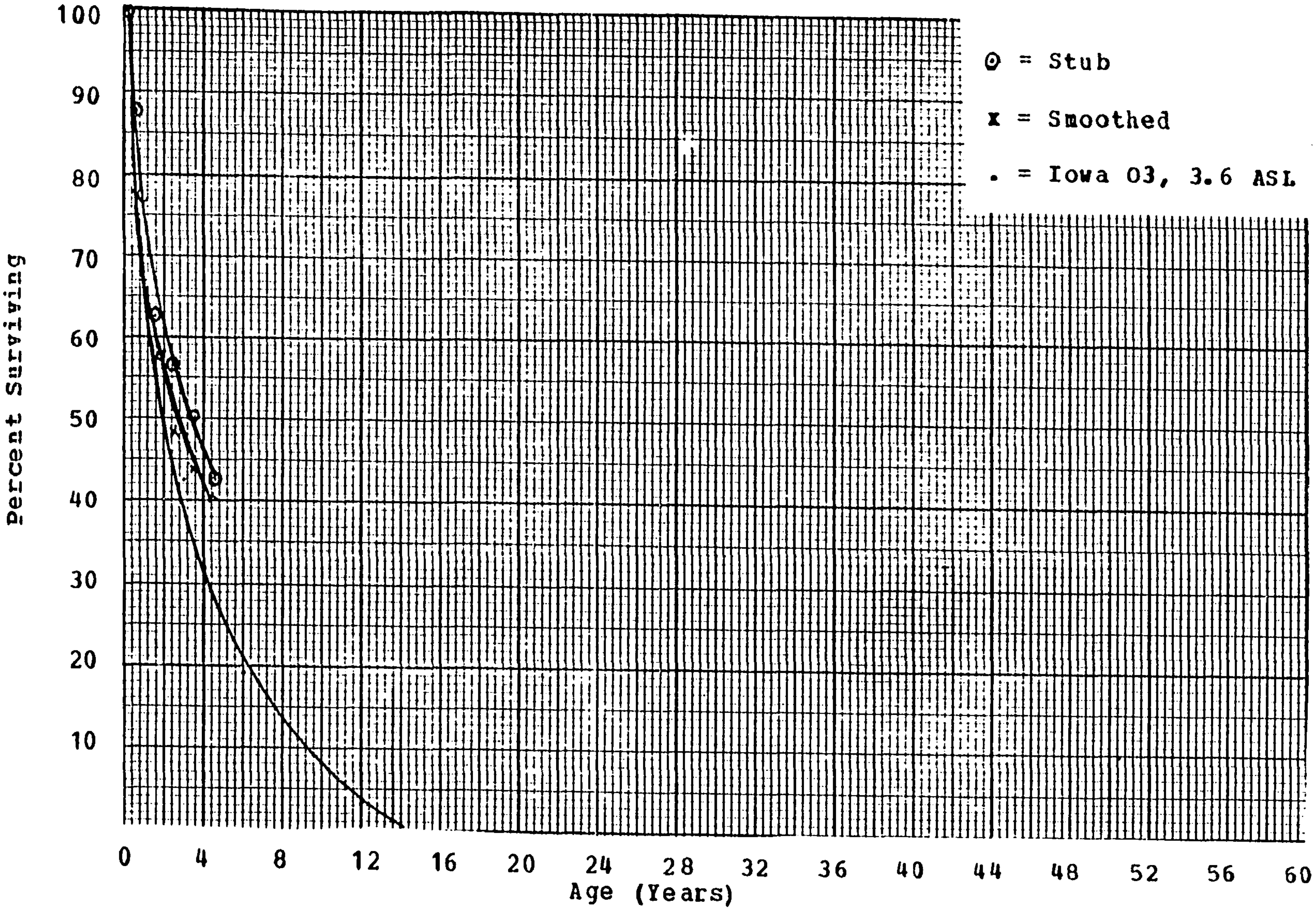

Figure 21. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Office, Career College Graduates, 1964-66 experience band

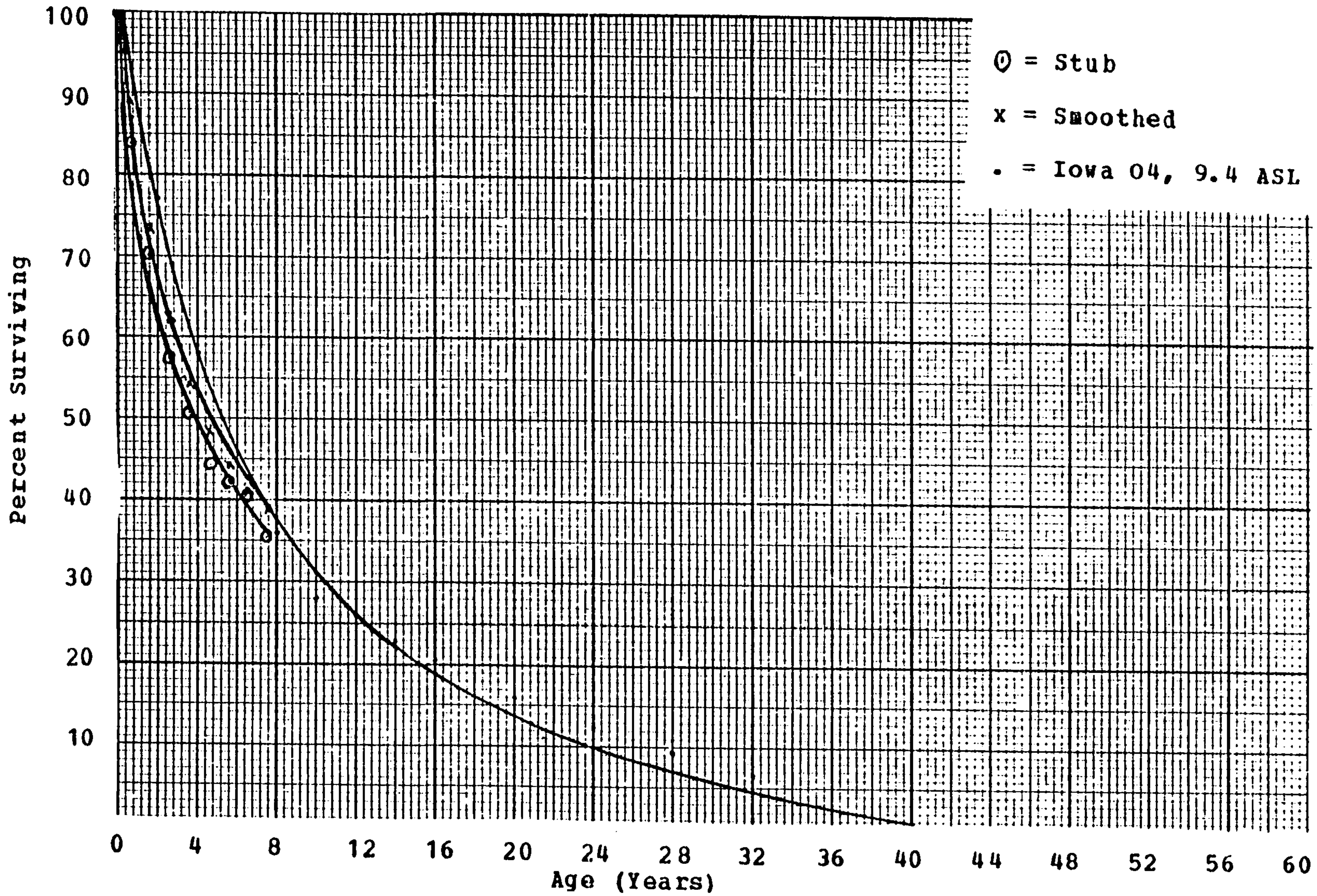

Figure 22. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Office, Career College Graduates, 1967-69 experience band

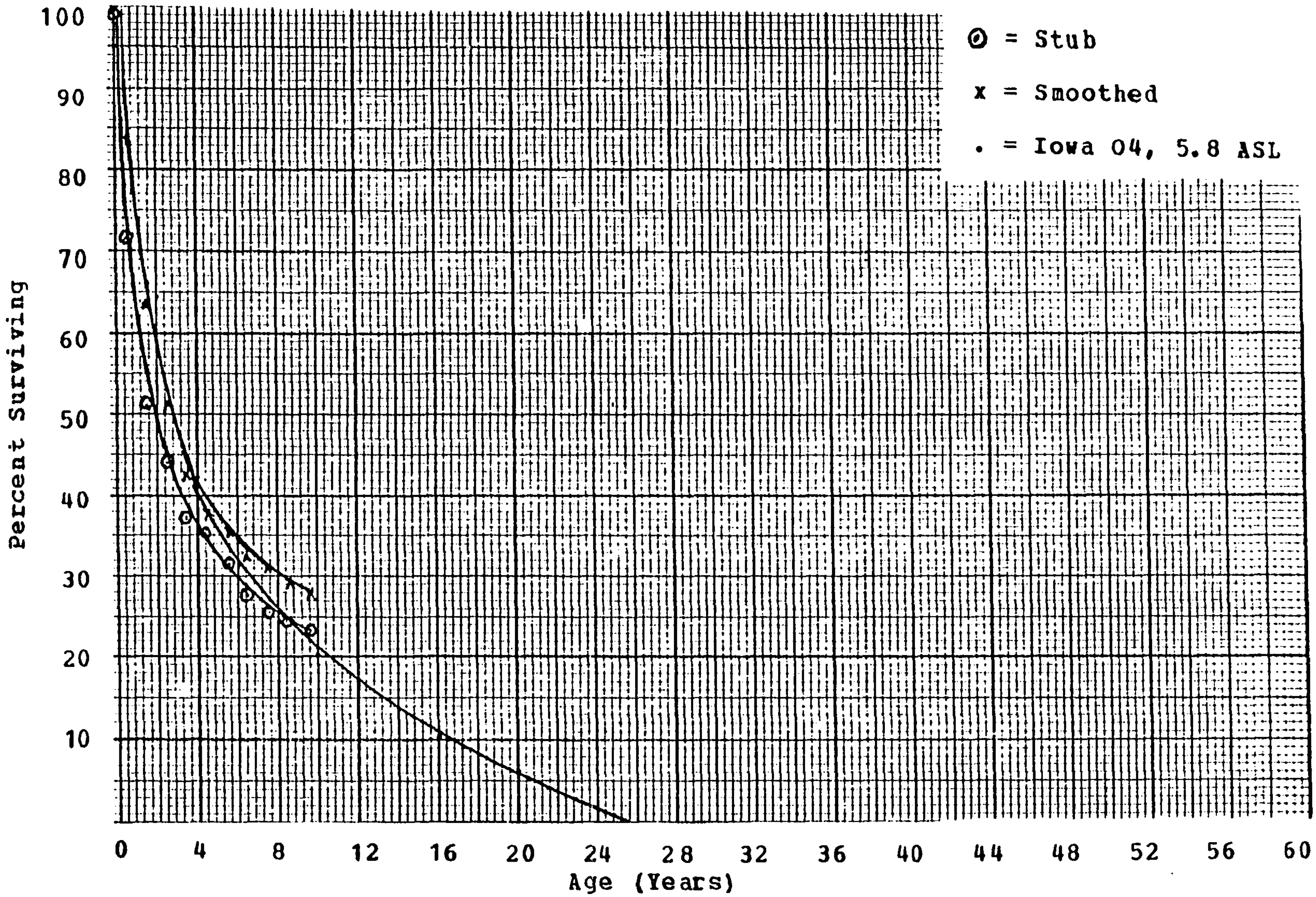

Figure 23. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Office, Career College Graduates, 1970-71 experience band

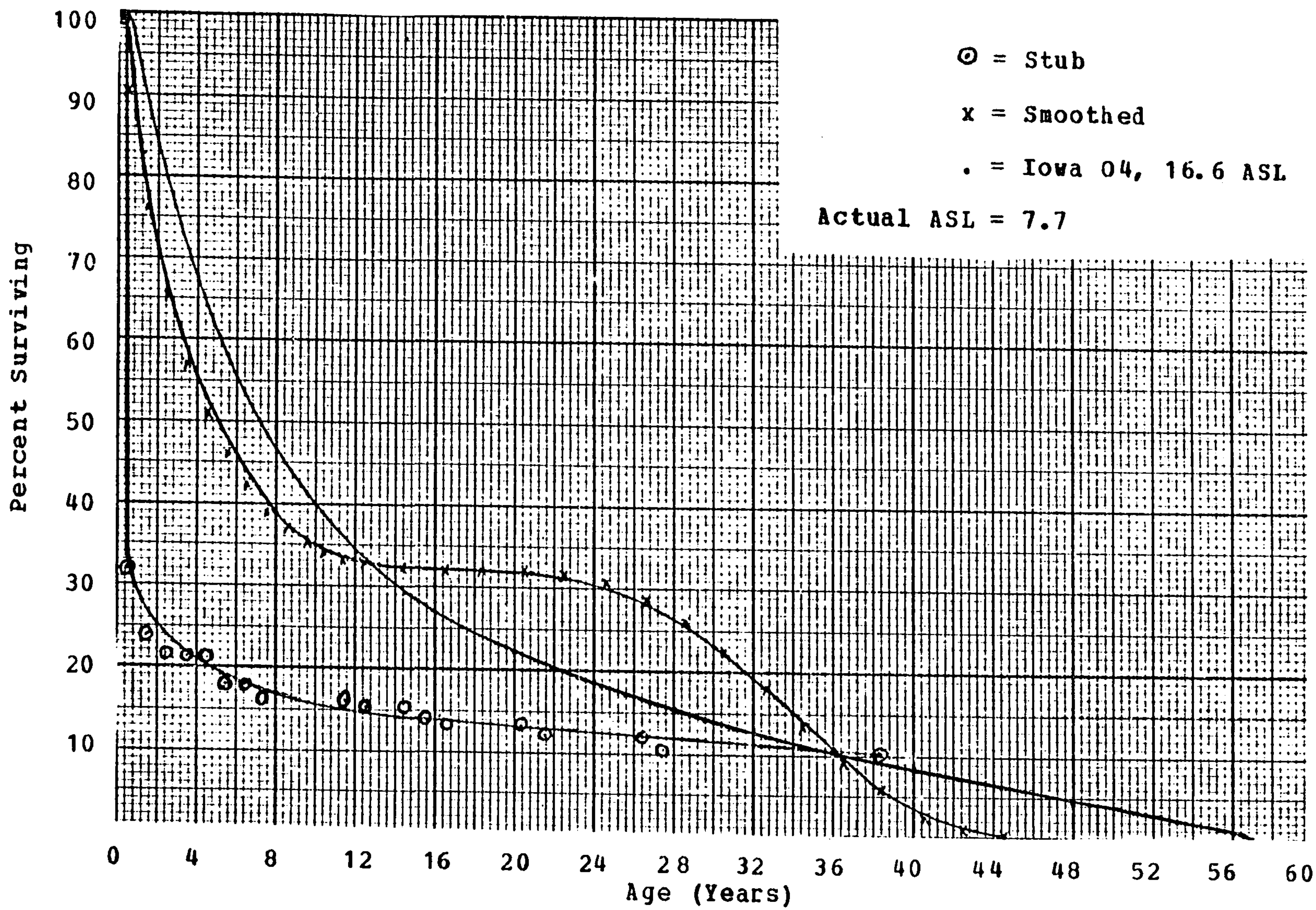

Figure 24. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Meter Readers, 1969-71 experience band

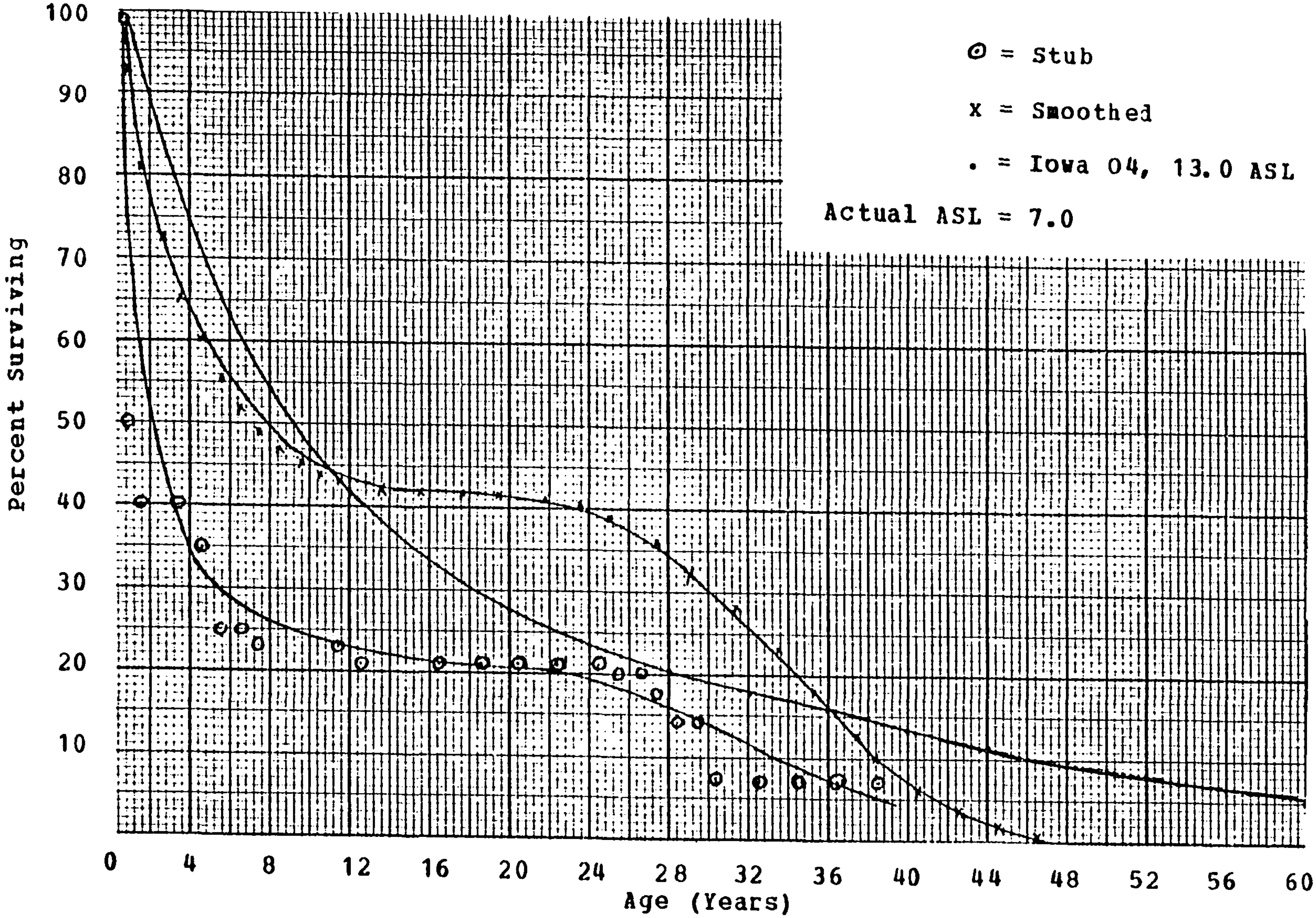

Figure 25. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Meter Readers, 1970-72 experience band

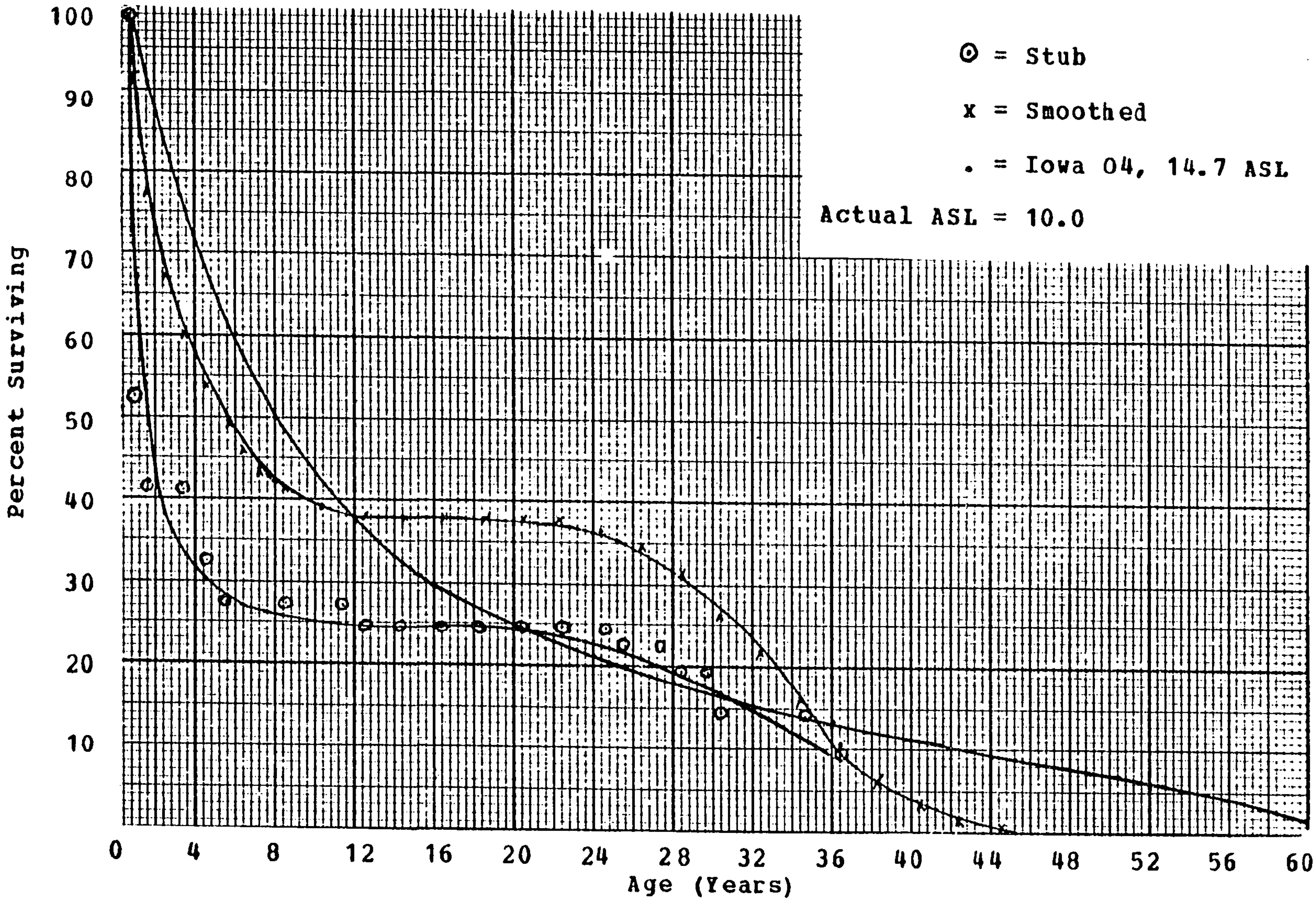

Figure 26. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Meter Readers, 1971-73 experience band

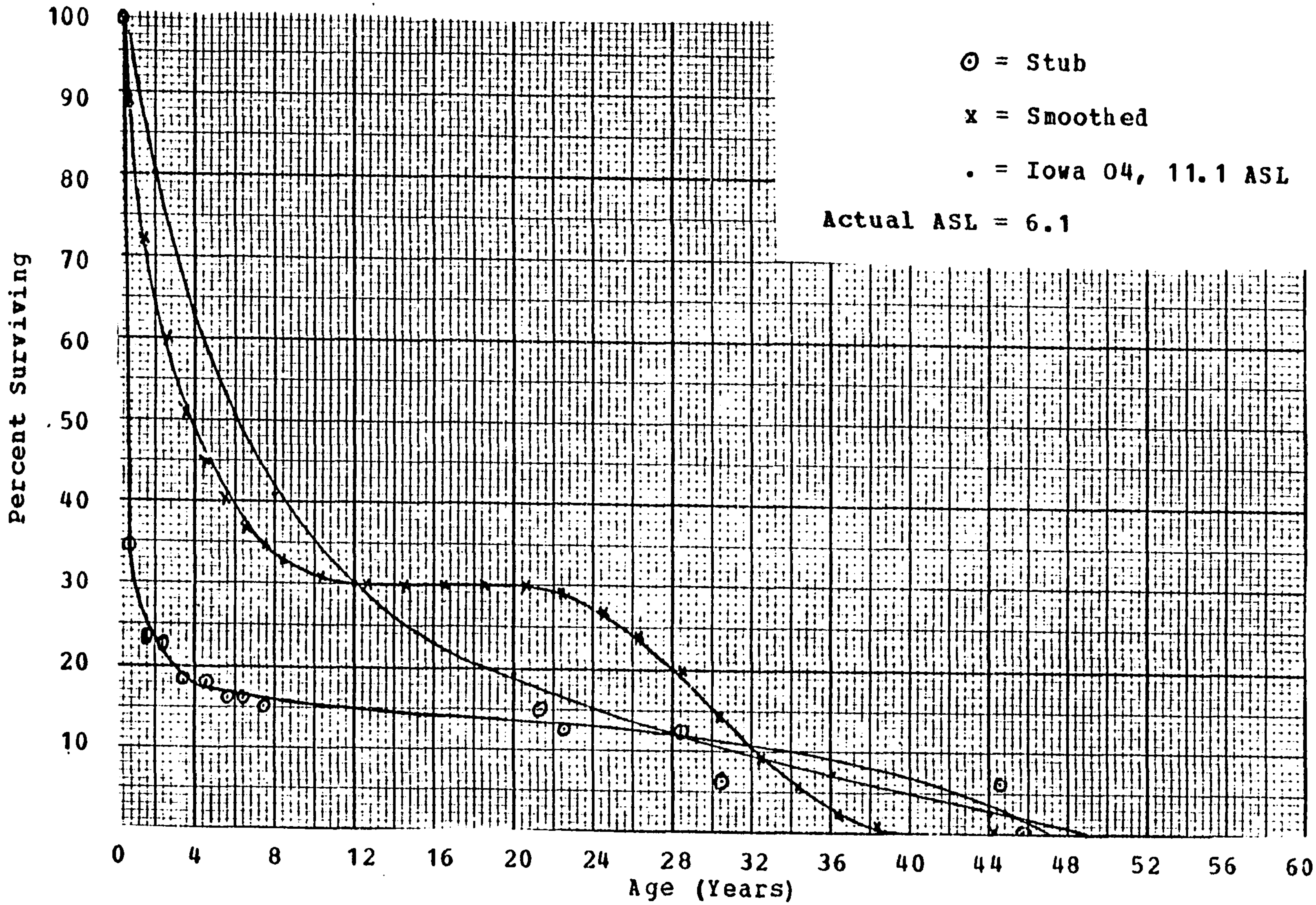

Figure 27. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Mechanics, 1969-71 experience band

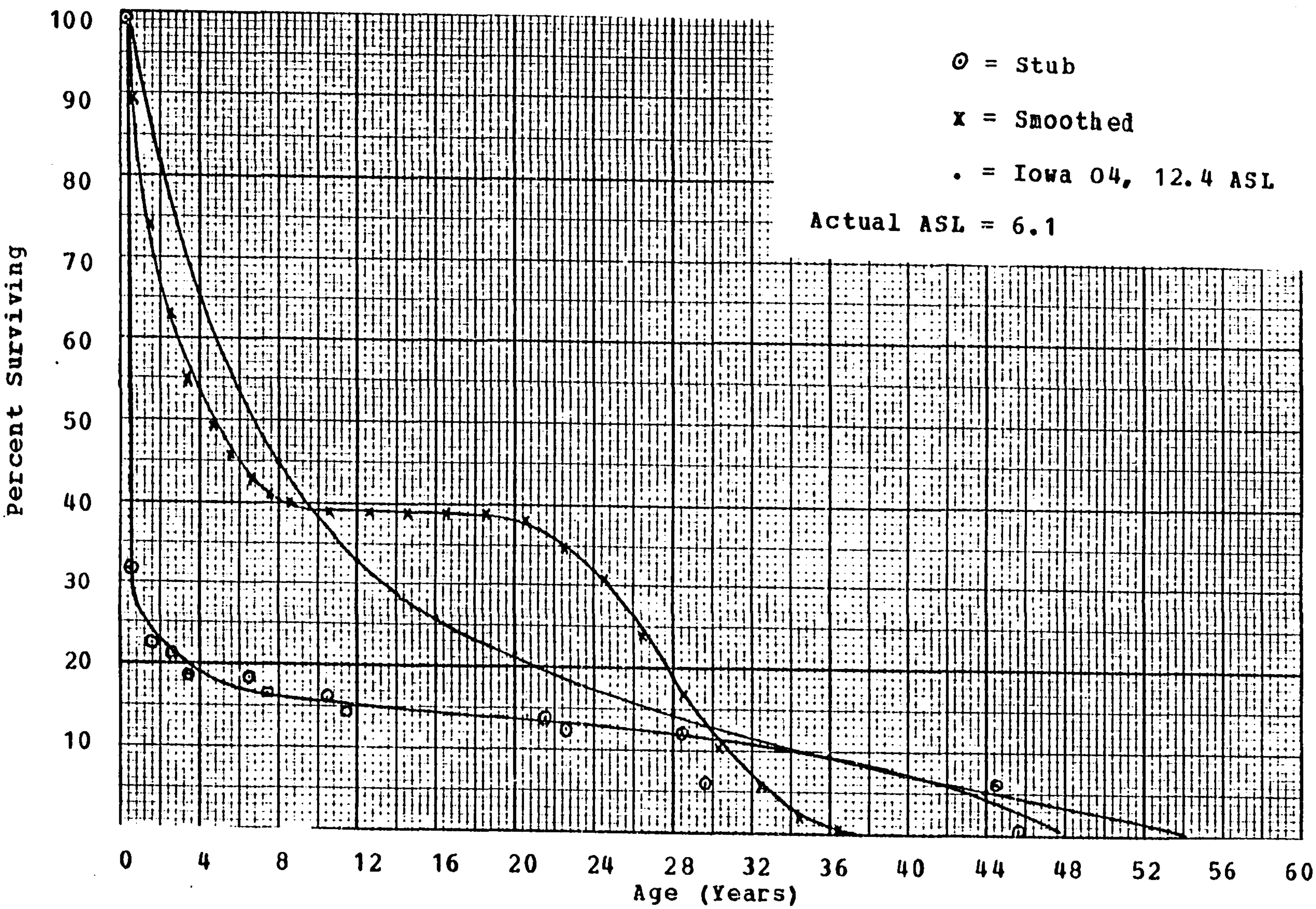

Figure 28. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Mechanics, 1970-72 experience band

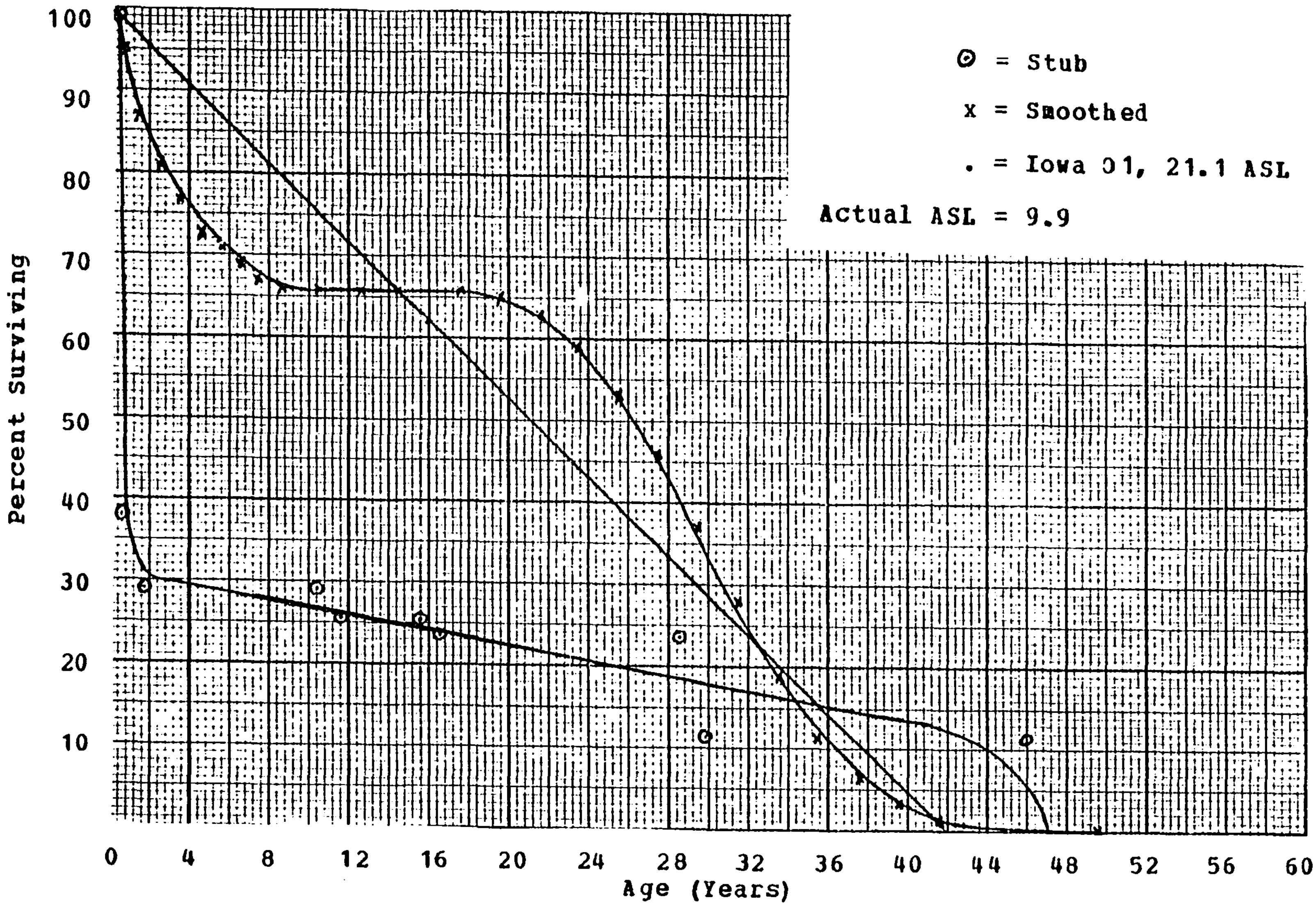

Figure 29. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Mechanics, 1971-73 experience band

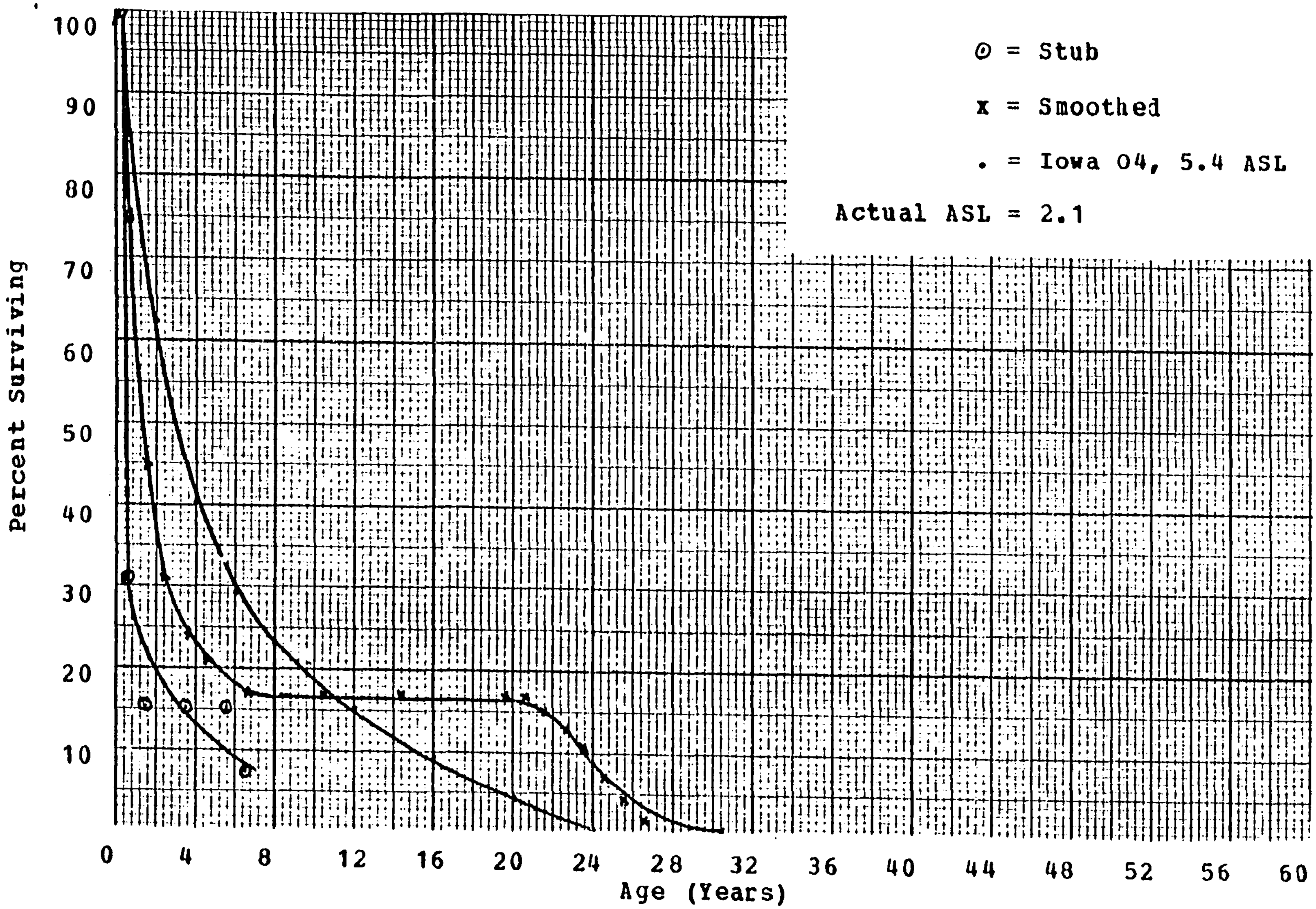

Figure 30. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Coal and Ash Handlers, 1969-71 experience band

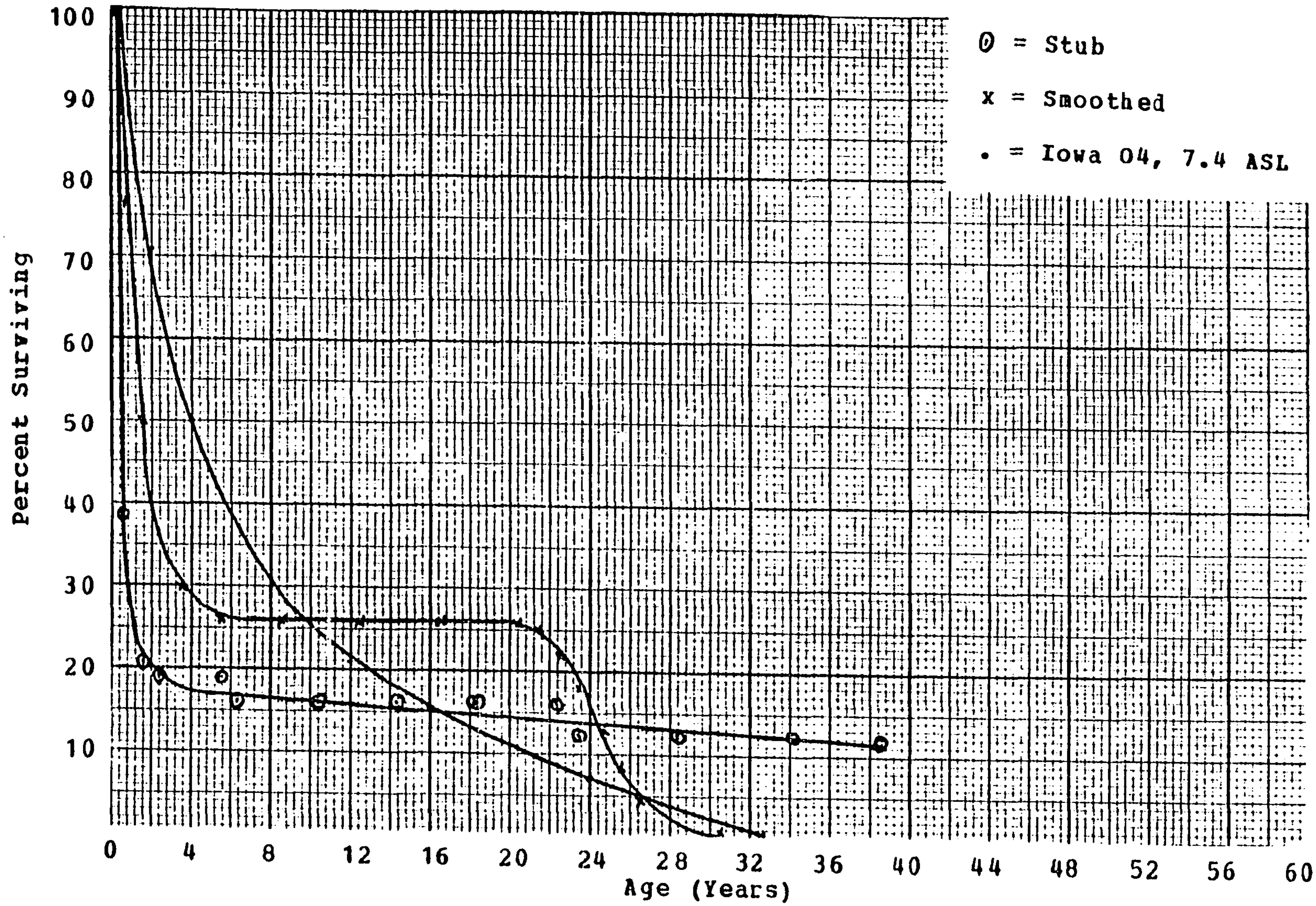

Figure 31. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Coal and Ash Handlers, 1970-72 experience band

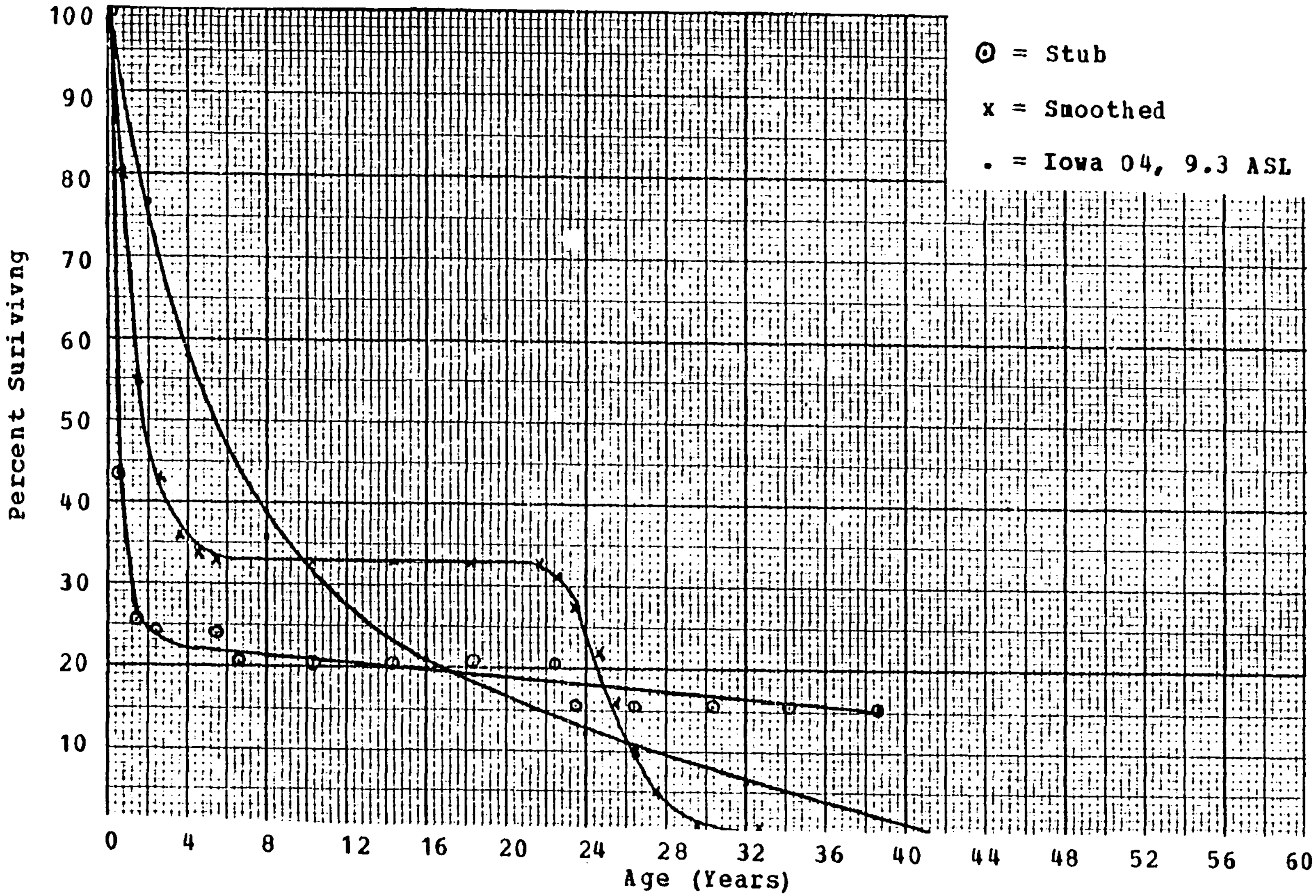

Figure 32. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Coal and Ash Handlers, 1971-73 experience band

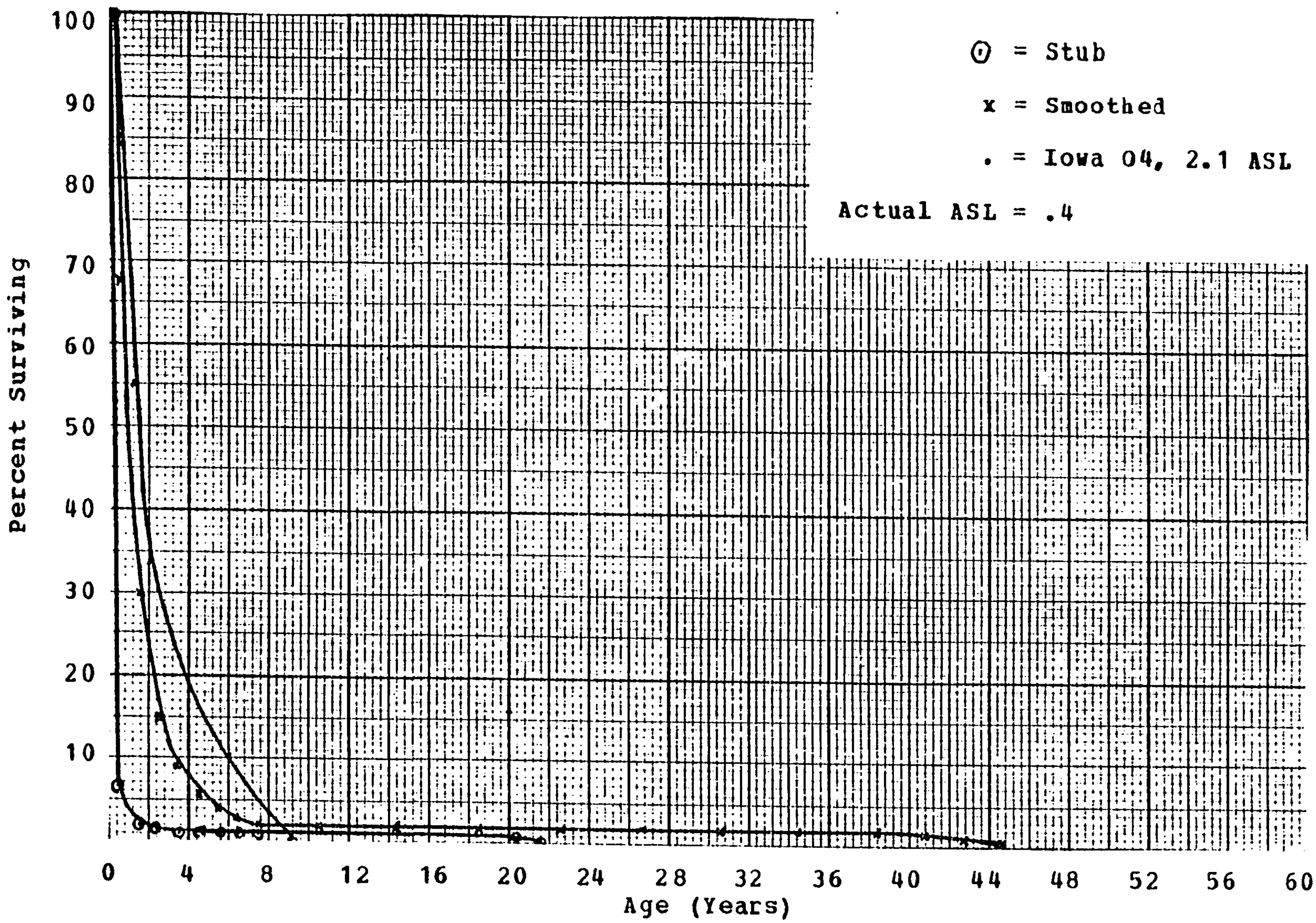

Figure 33. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Laborers, 1969-71 experience band

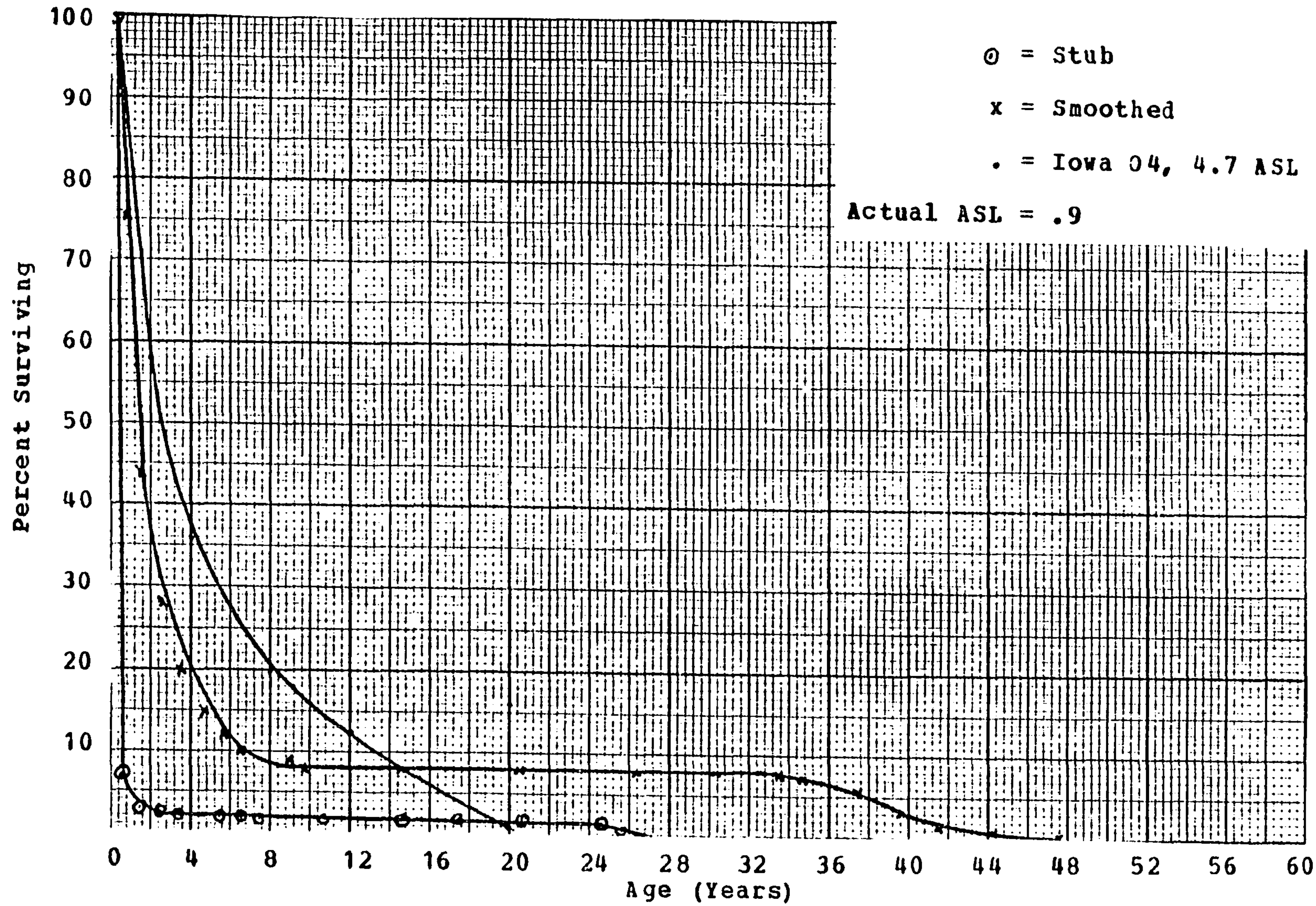

Figure 34. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Laborers, 1970-72 experience band

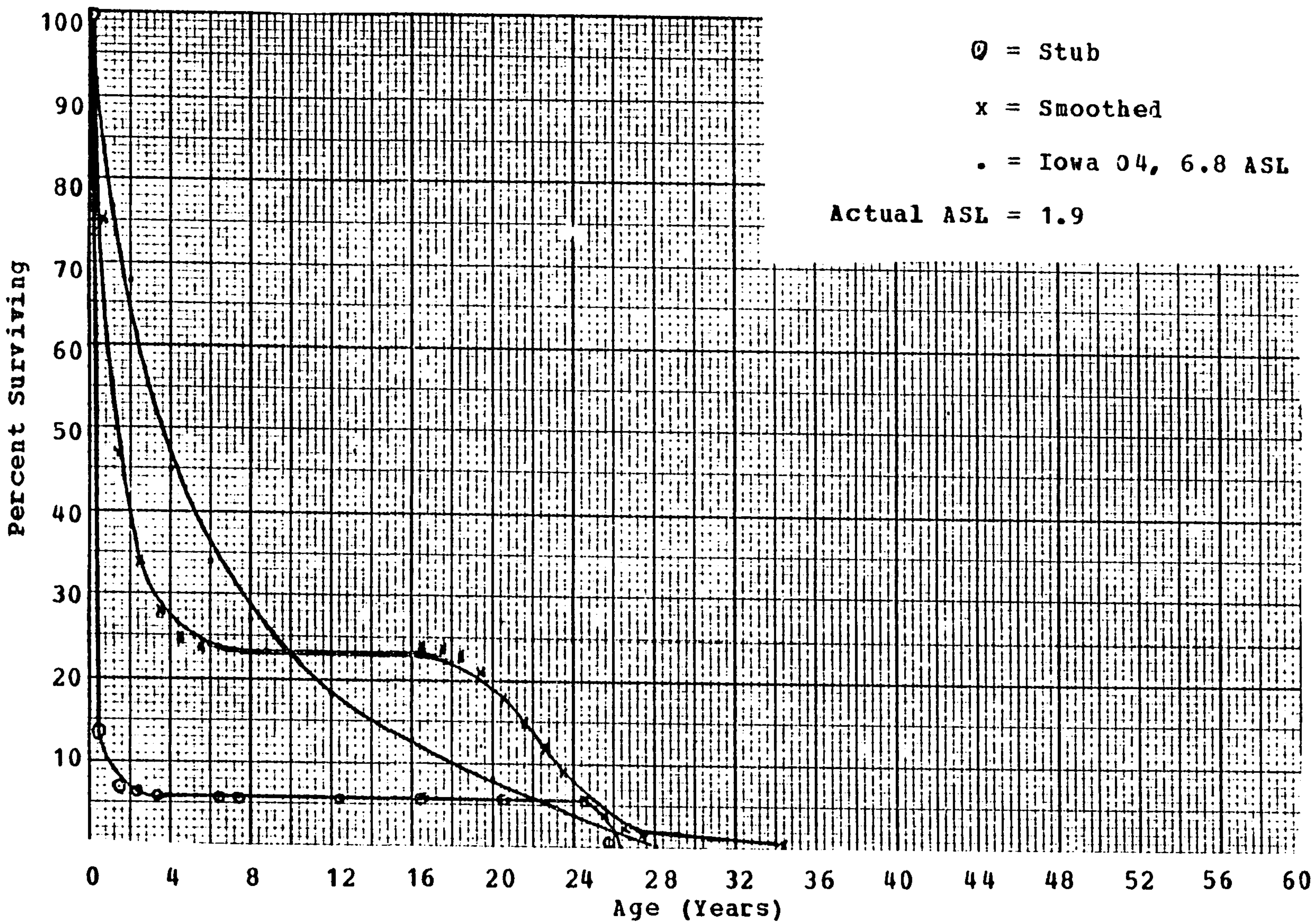

Figure 35. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Laborers, 1971-73 experience band

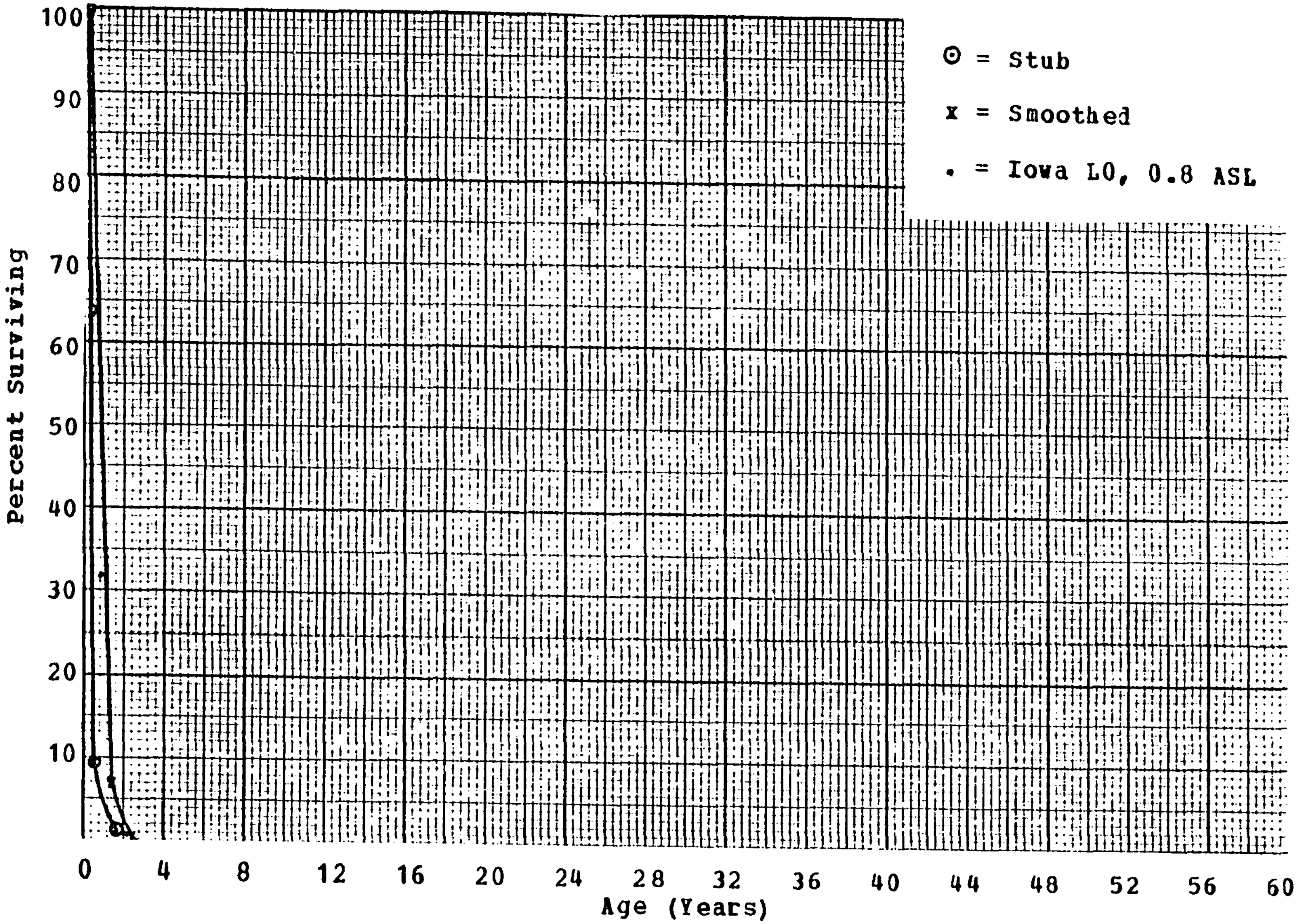

Figure 36. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Ironworkers, 1969-71 experience band

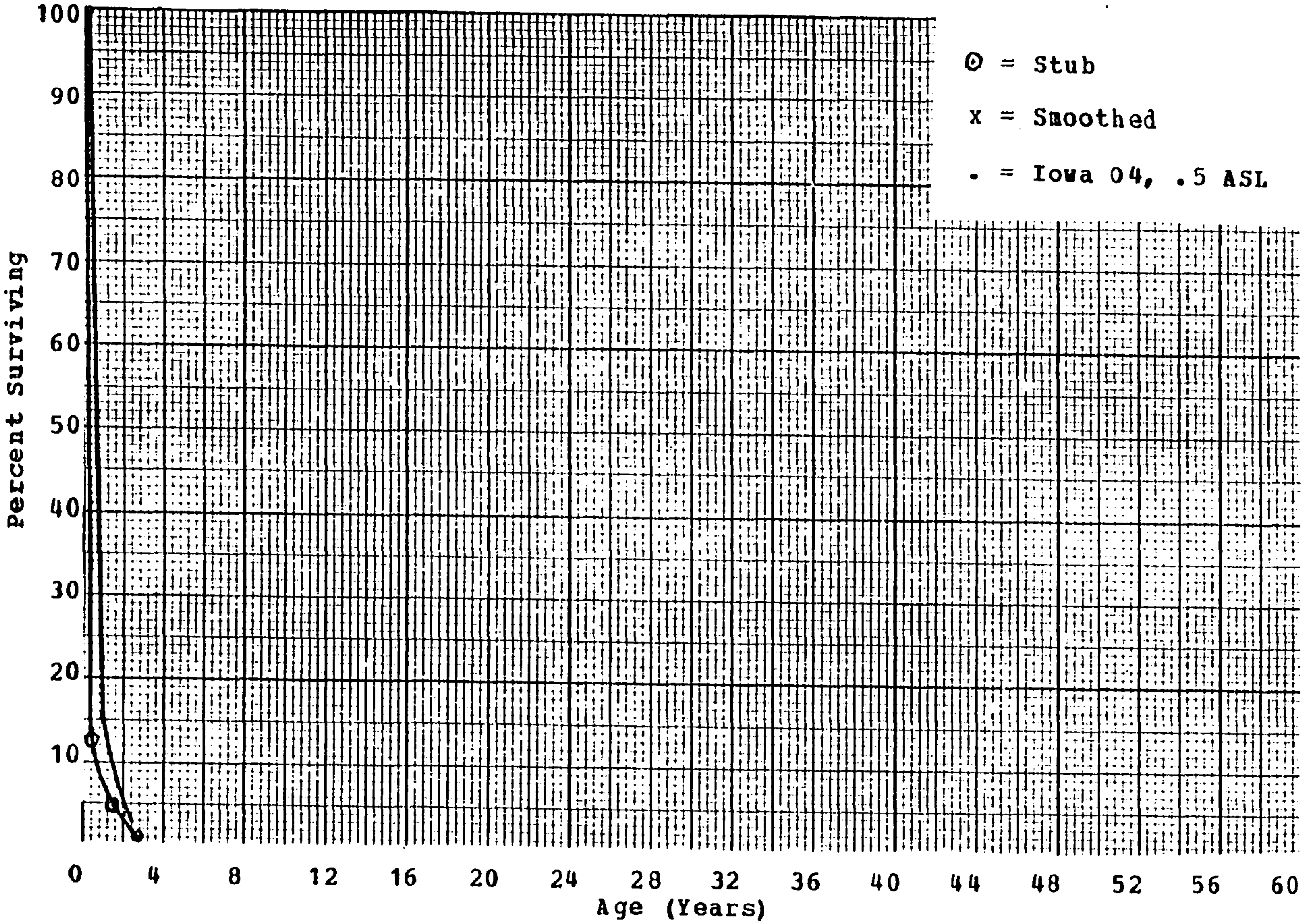

Figure 37. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Ironworkers, 1970-72 experience band

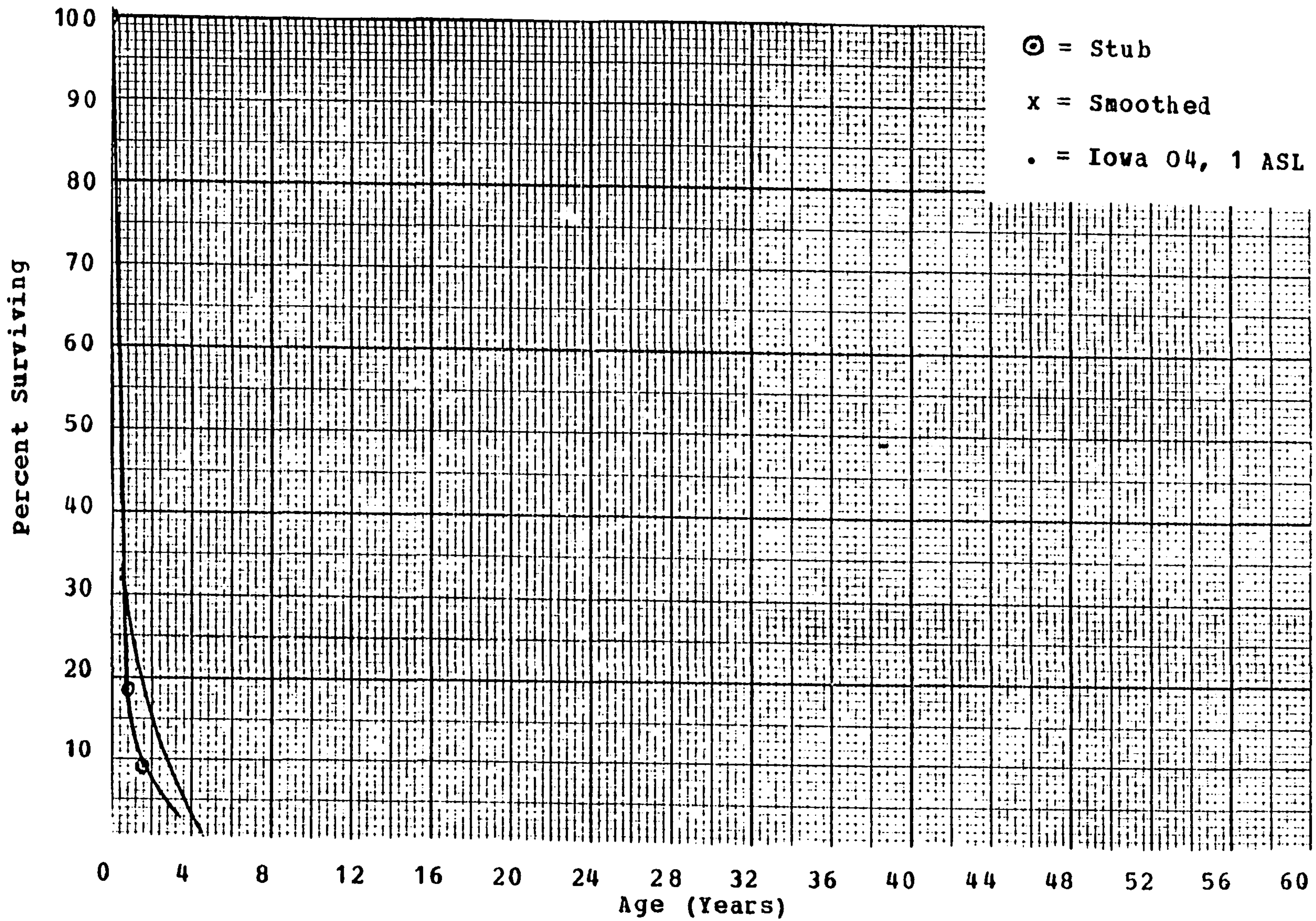

Figure 38. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Ironworkers, 1971-73 experience band

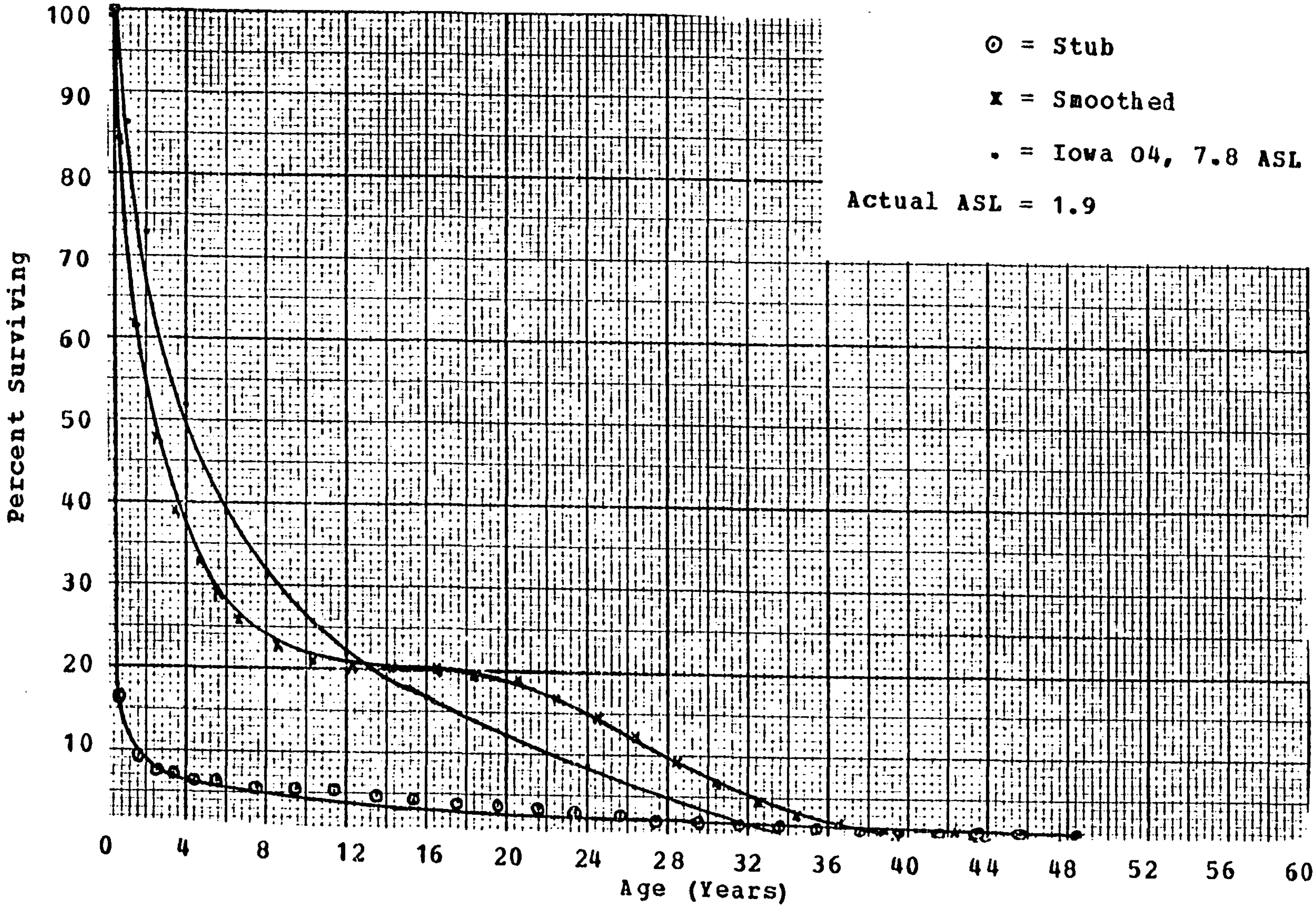

Figure 39. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Union Employees, 1969-71 experience band

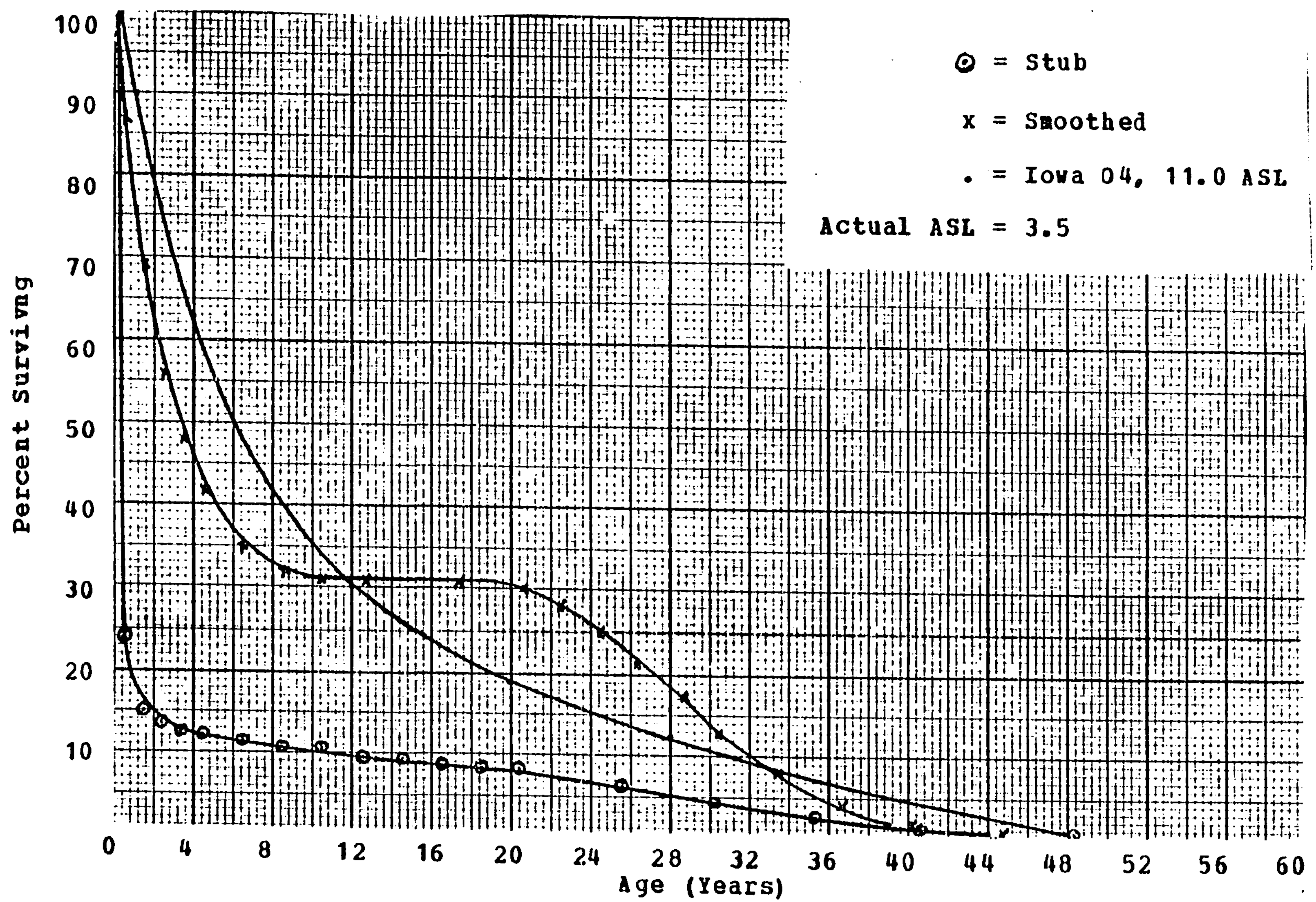

Figure 40. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Union Employees, 1970-72 experience band

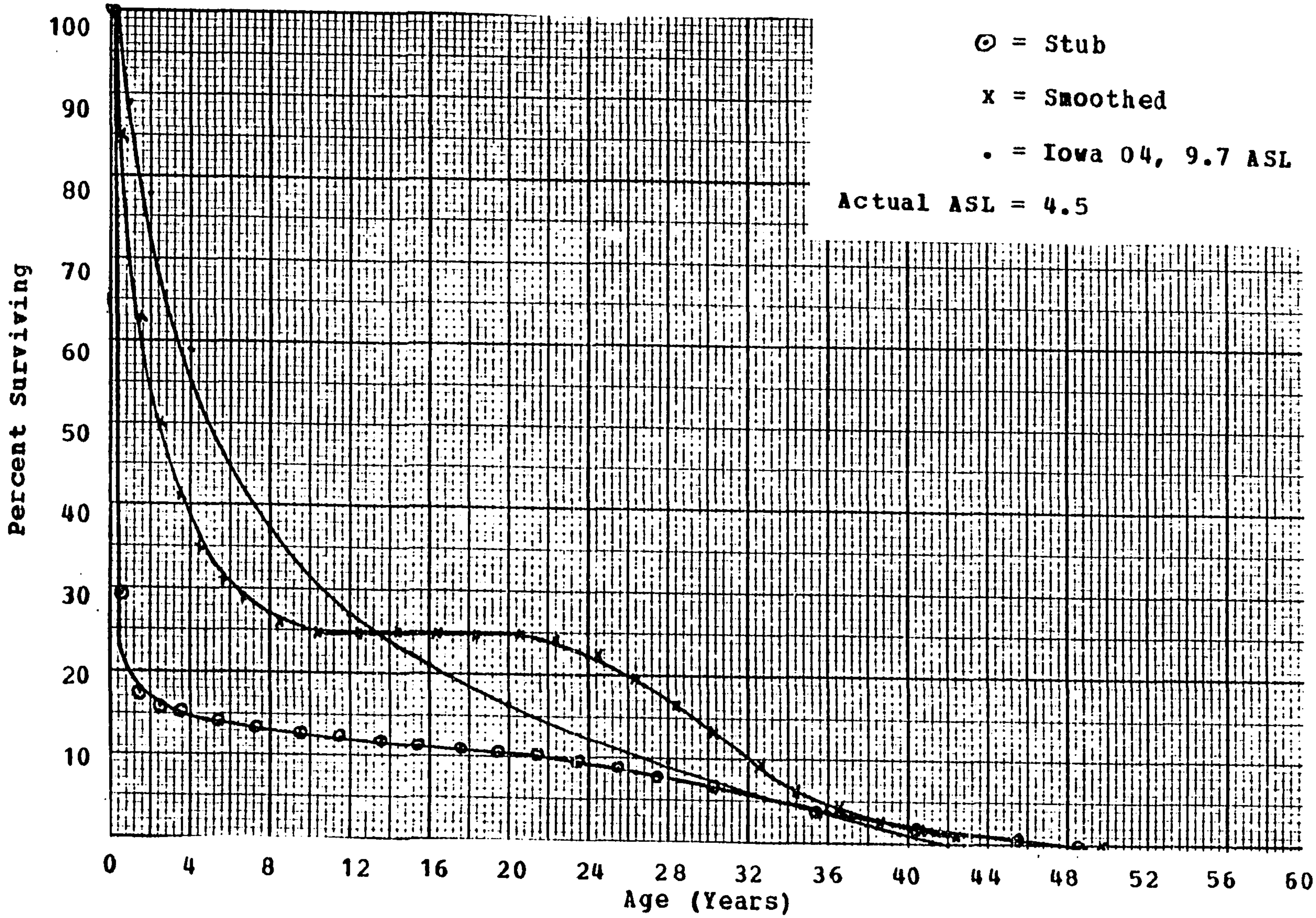

Figure 41. Comparison of stub survivor curve, smoothed curve, and Iowa curve, Utility Union Employees, 1971-73 experience band

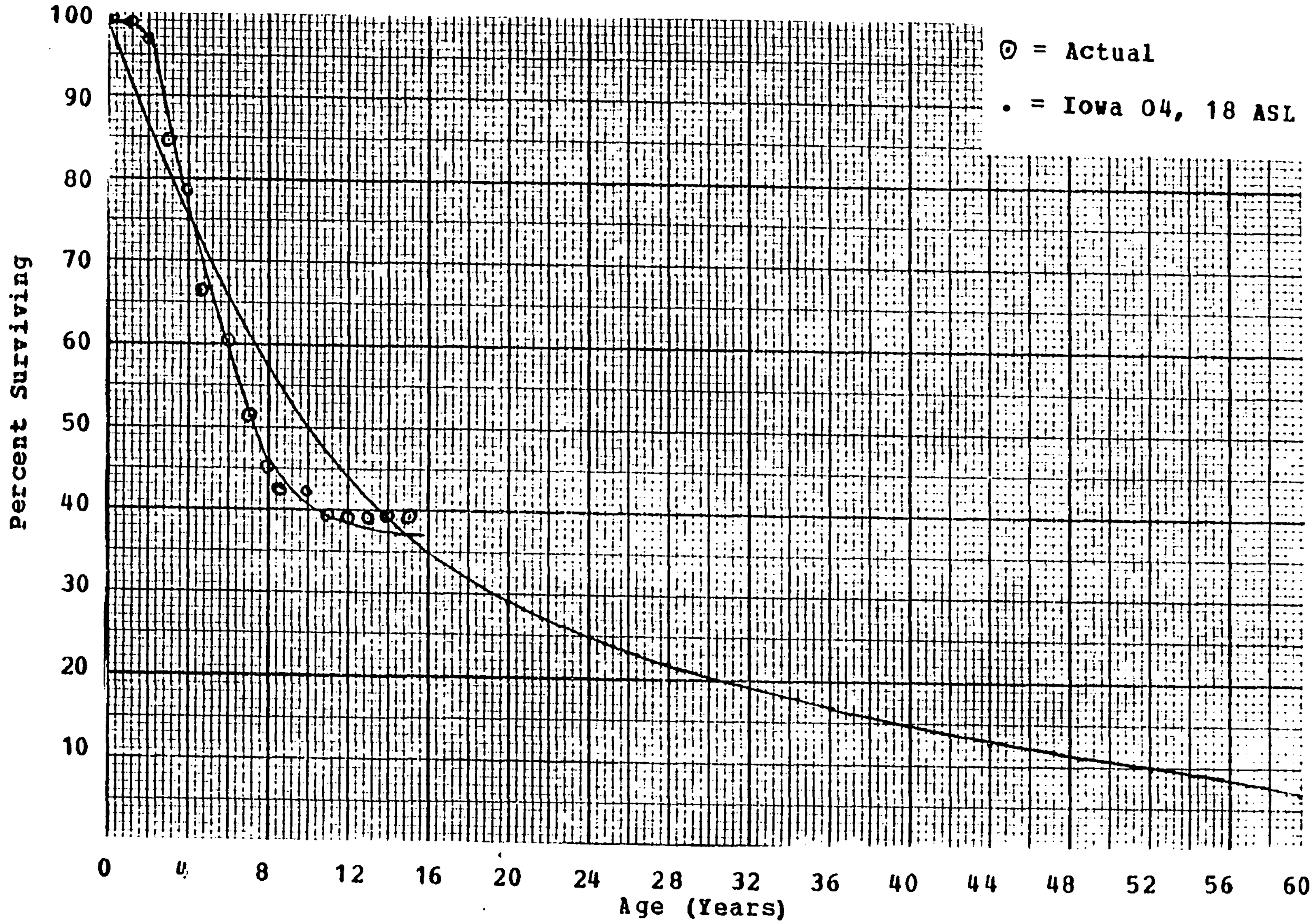

Figure 42. Manufacturing Marketing, original group, 1958-60

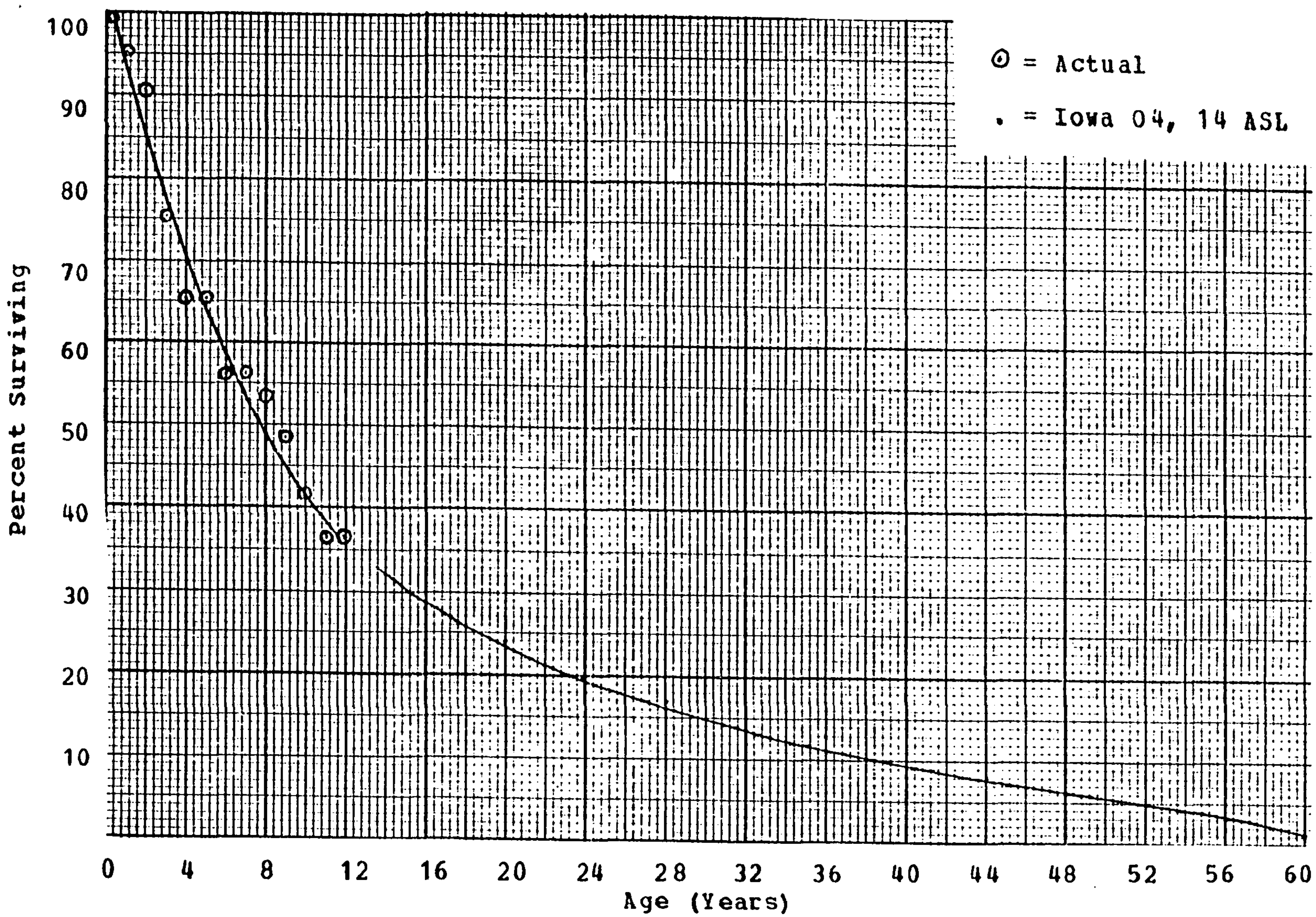

Figure 43. Manufacturing Marketing, original group, 1961-63

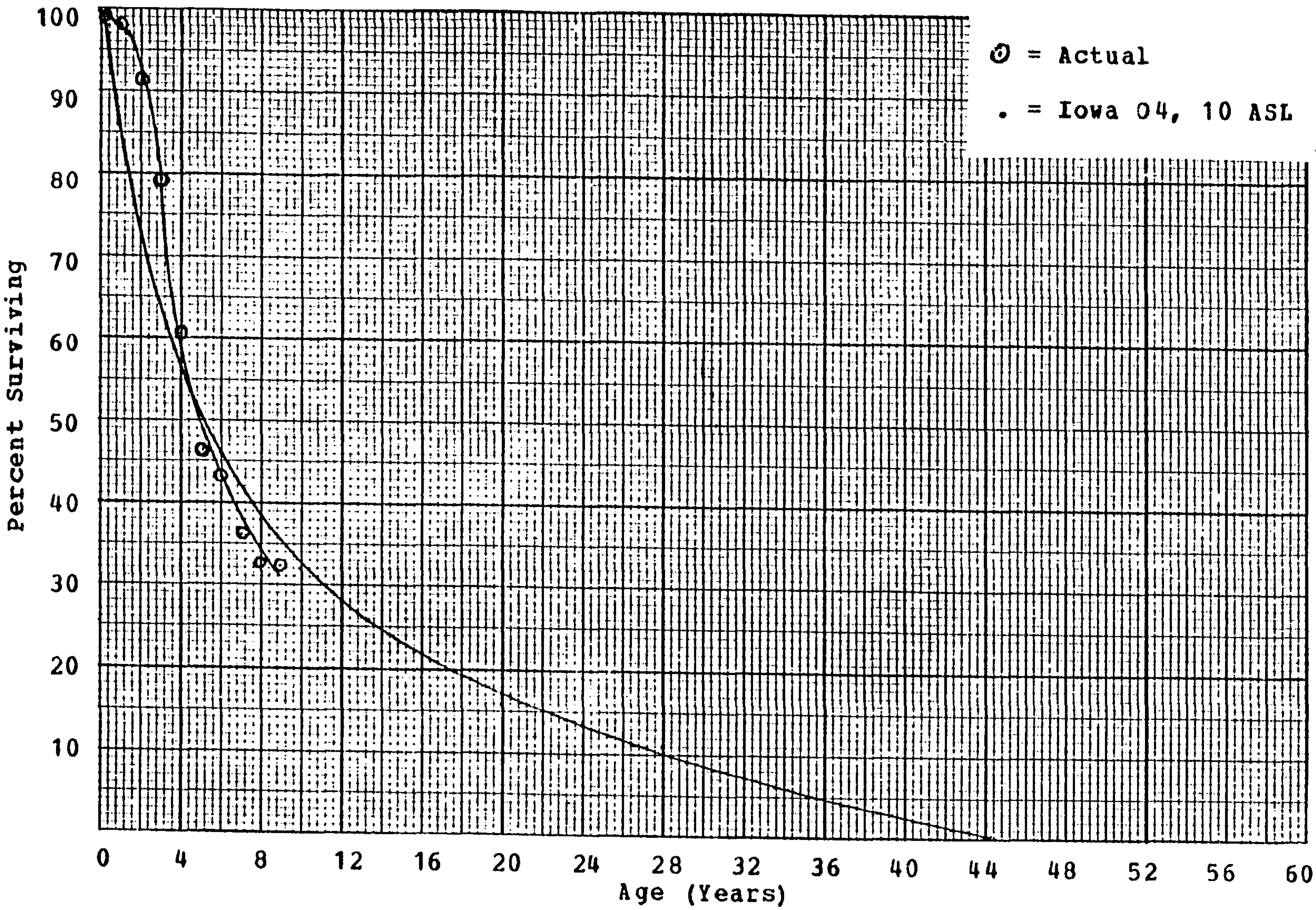

Figure 44. Manufacturing Marketing, original group, 1964-66

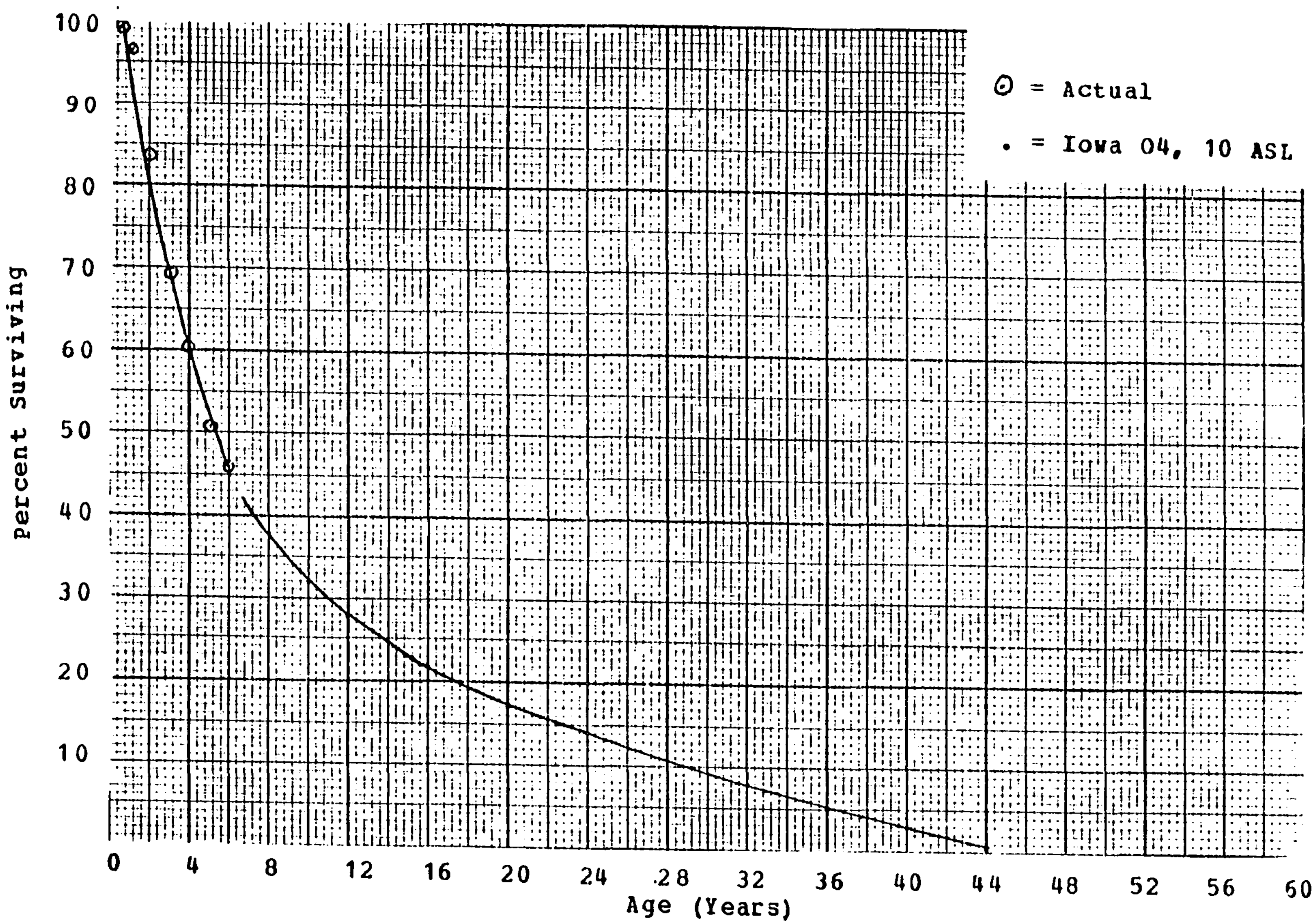

Figure 45. Manufacturing Marketing, original group, 1967-69.

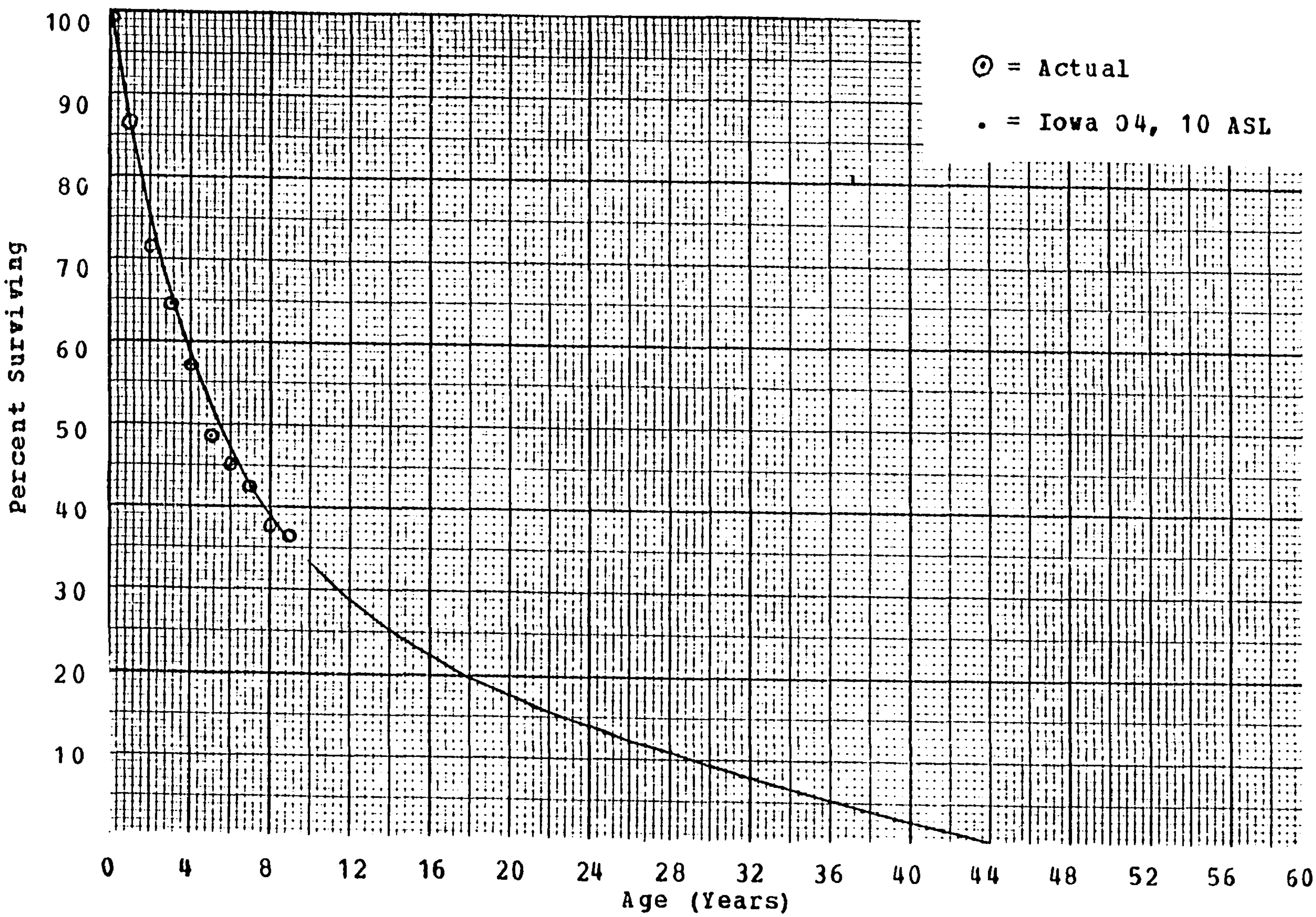

Figure 46. Office Career College Graduates, original group, 1961-63

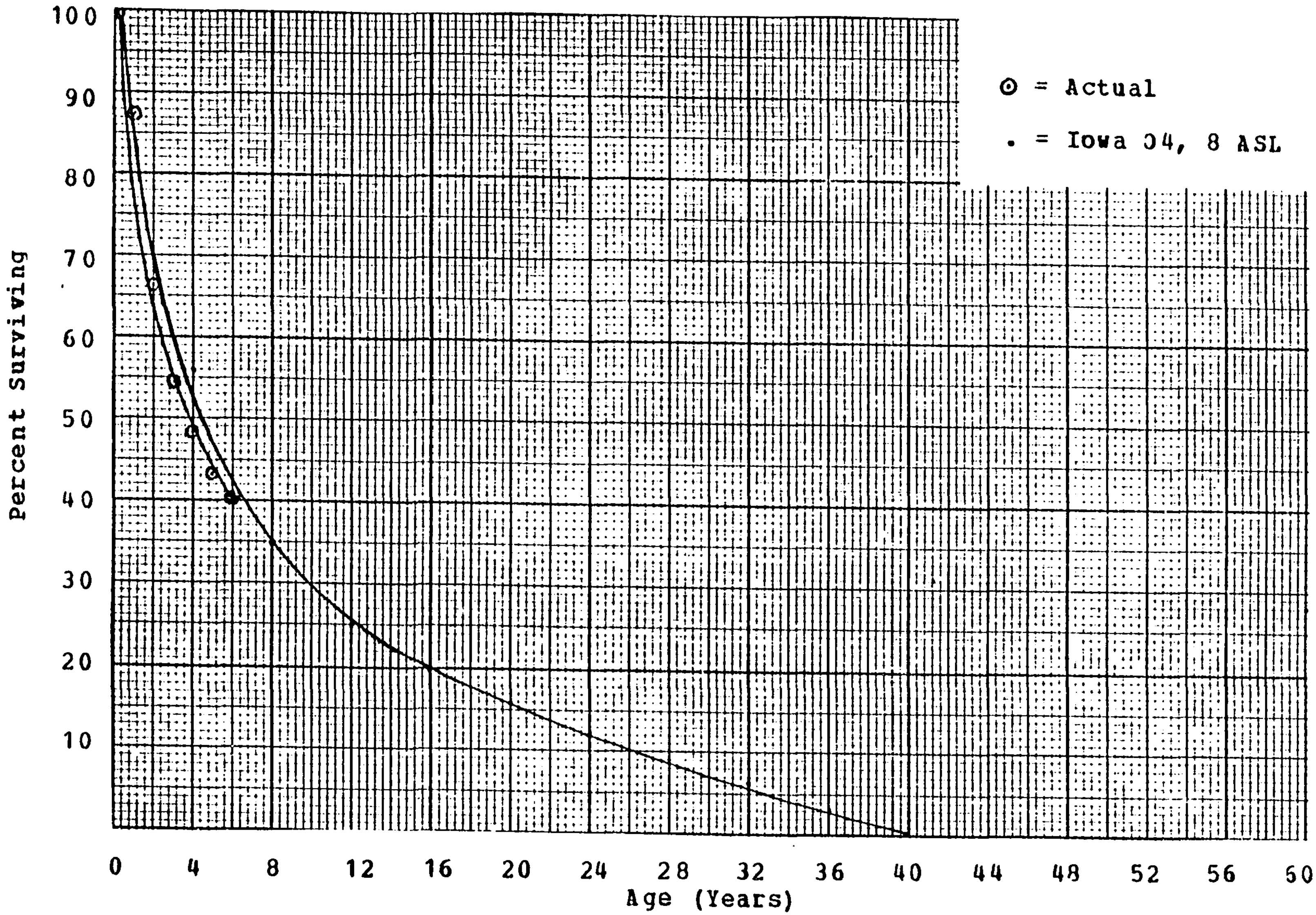

Figure 47. Office Career College Graduates, original group, 1964-66.

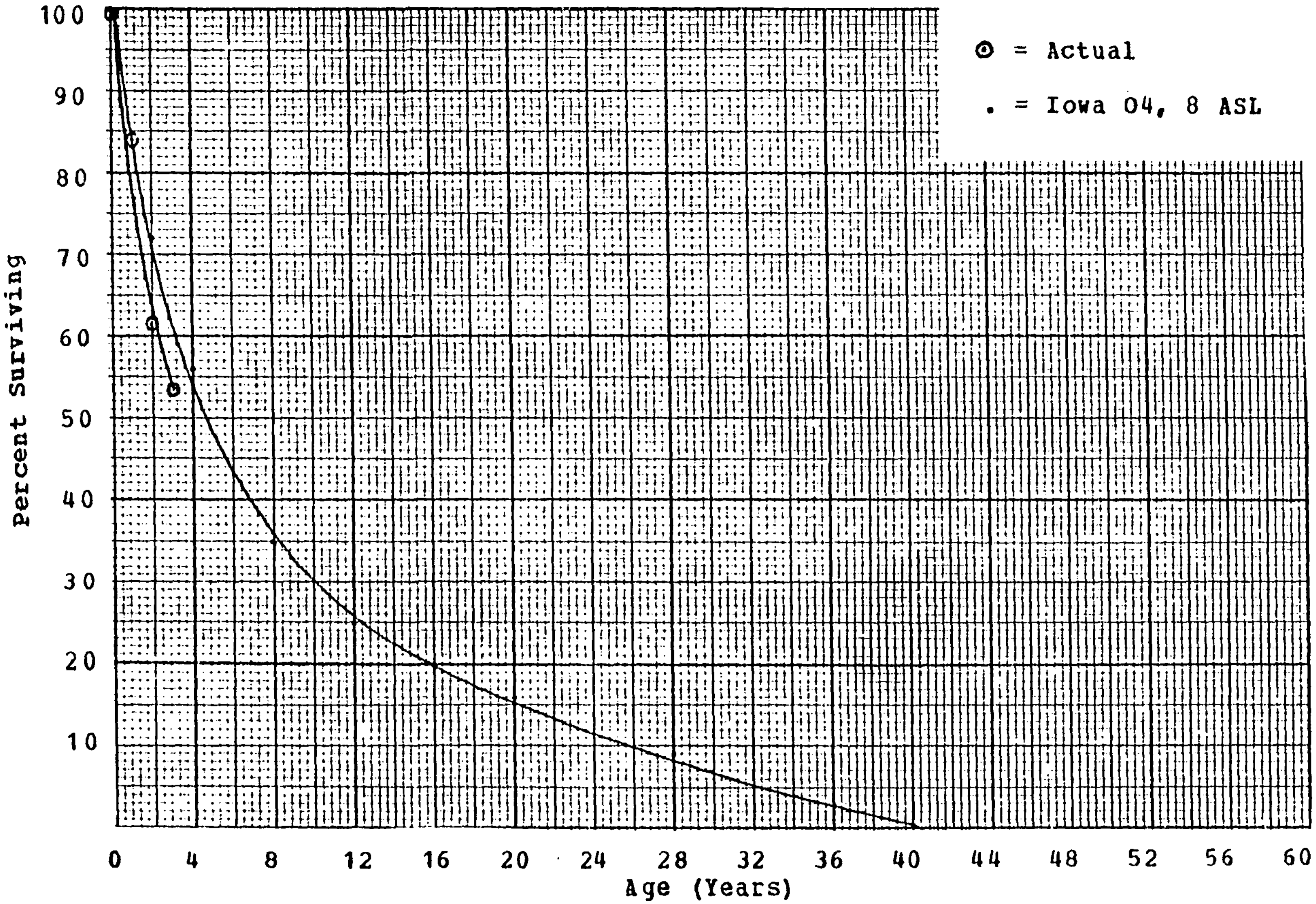

Figure 48. Office Career College Graduates, original group, 1967-69

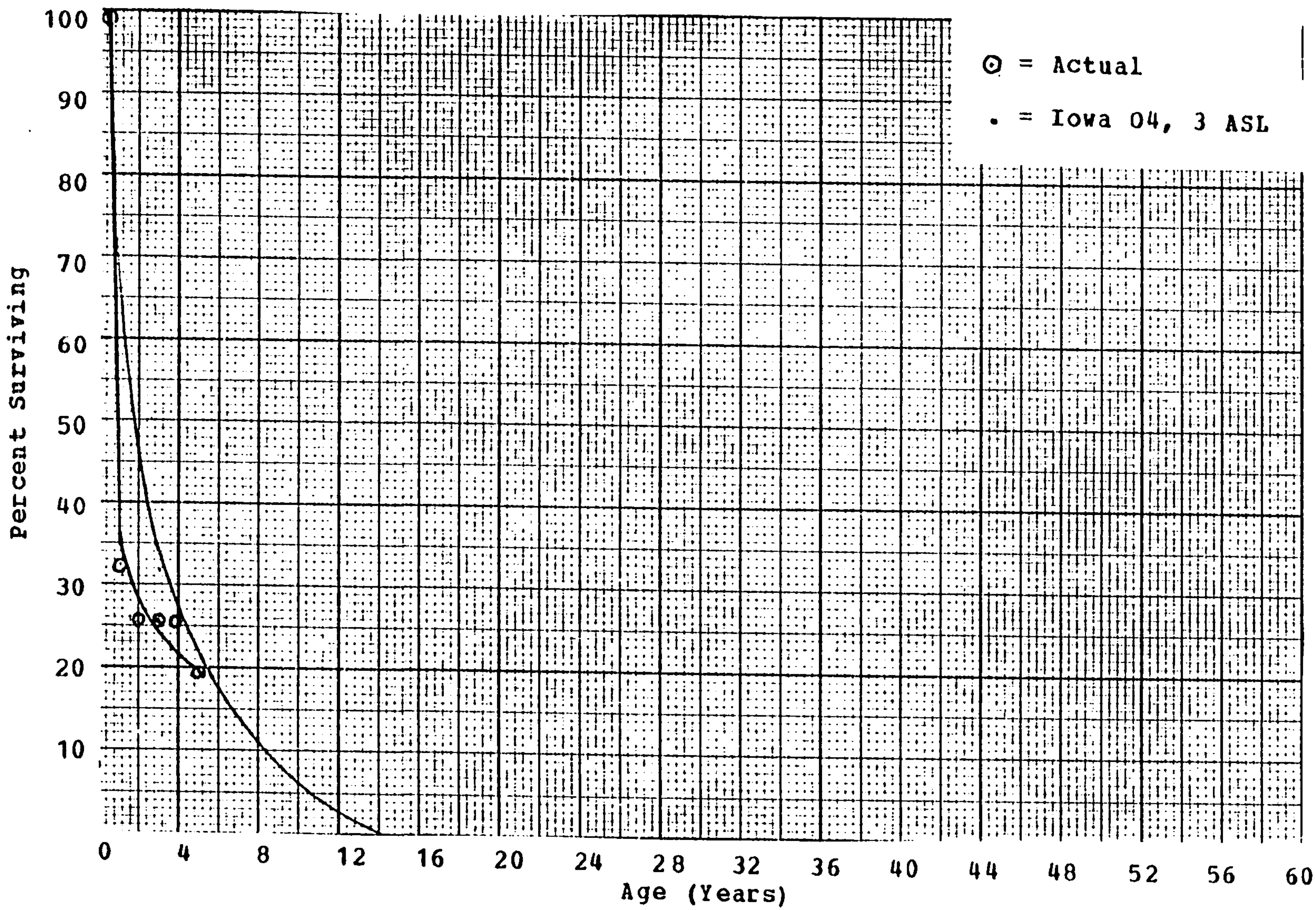

Figure 49. Utility Meter Readers, original group, 1969-71

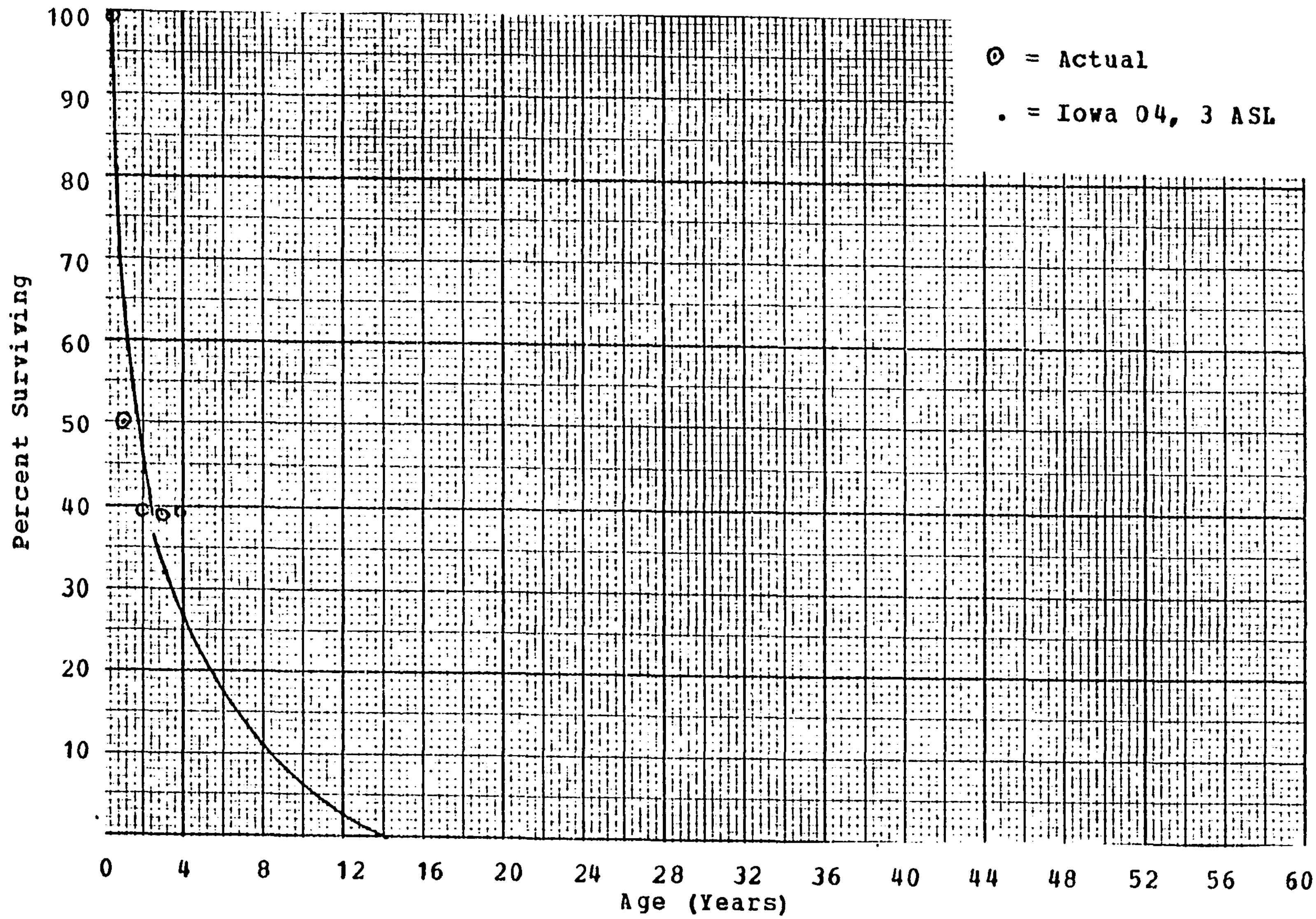

Figure 50. Utility Meter Readers, original group, 1970-72

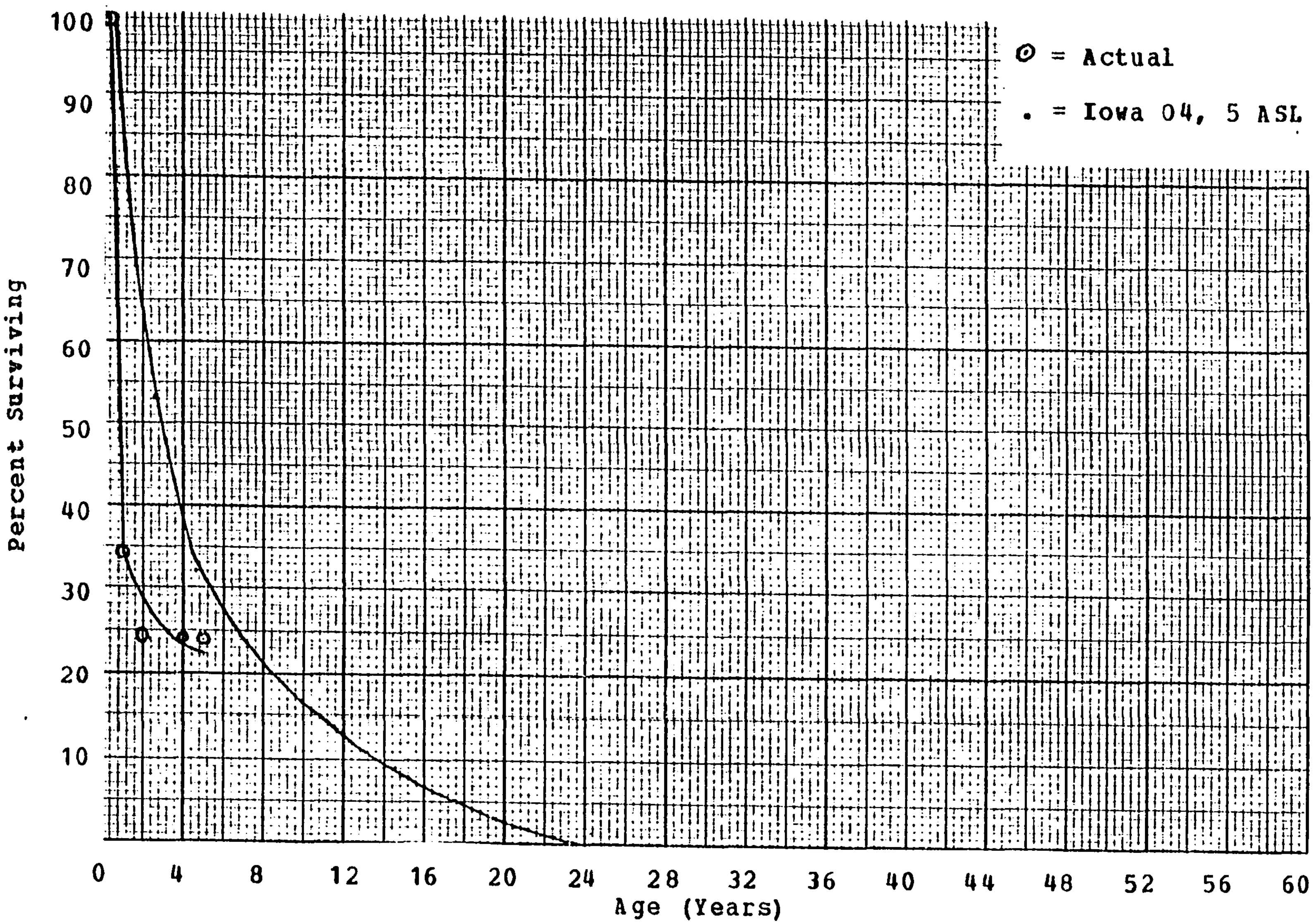

Figure 51. Utility Mechanics, original group, 1969-71

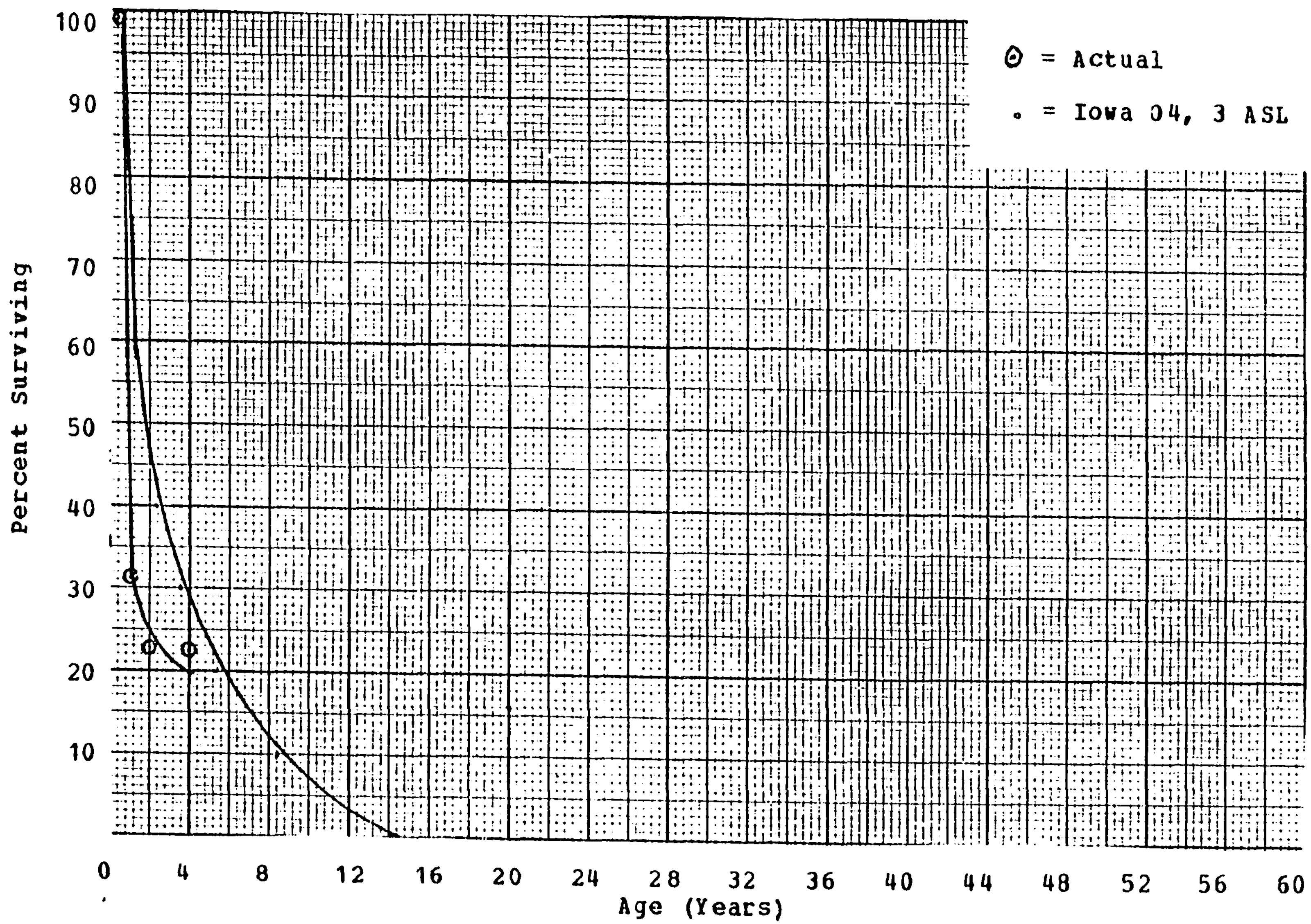

Figure 52. Utility Mechanics, original group, 1970-72

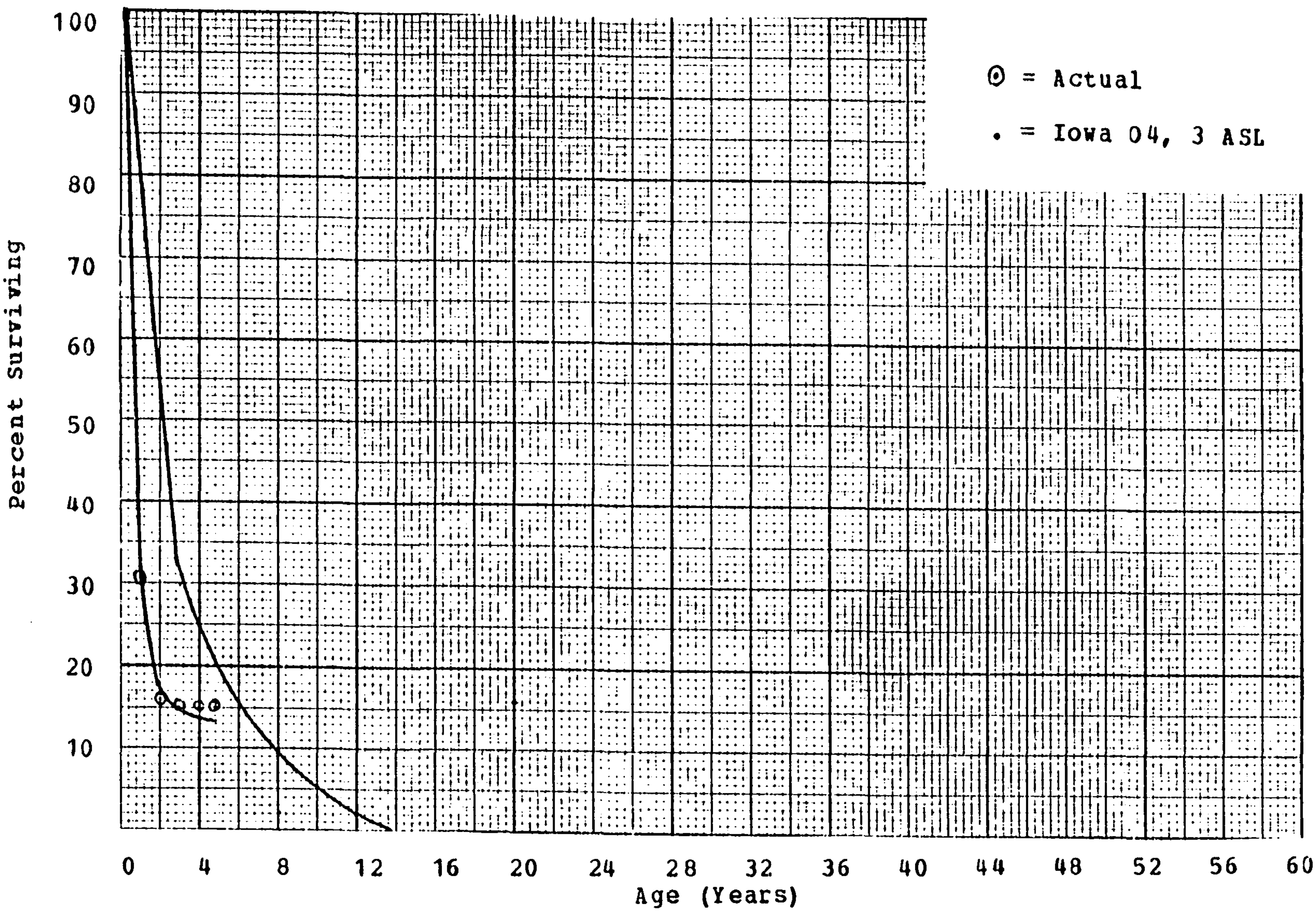

Figure 53. Utility Coal and Ash Handlers, original group, 1969-71

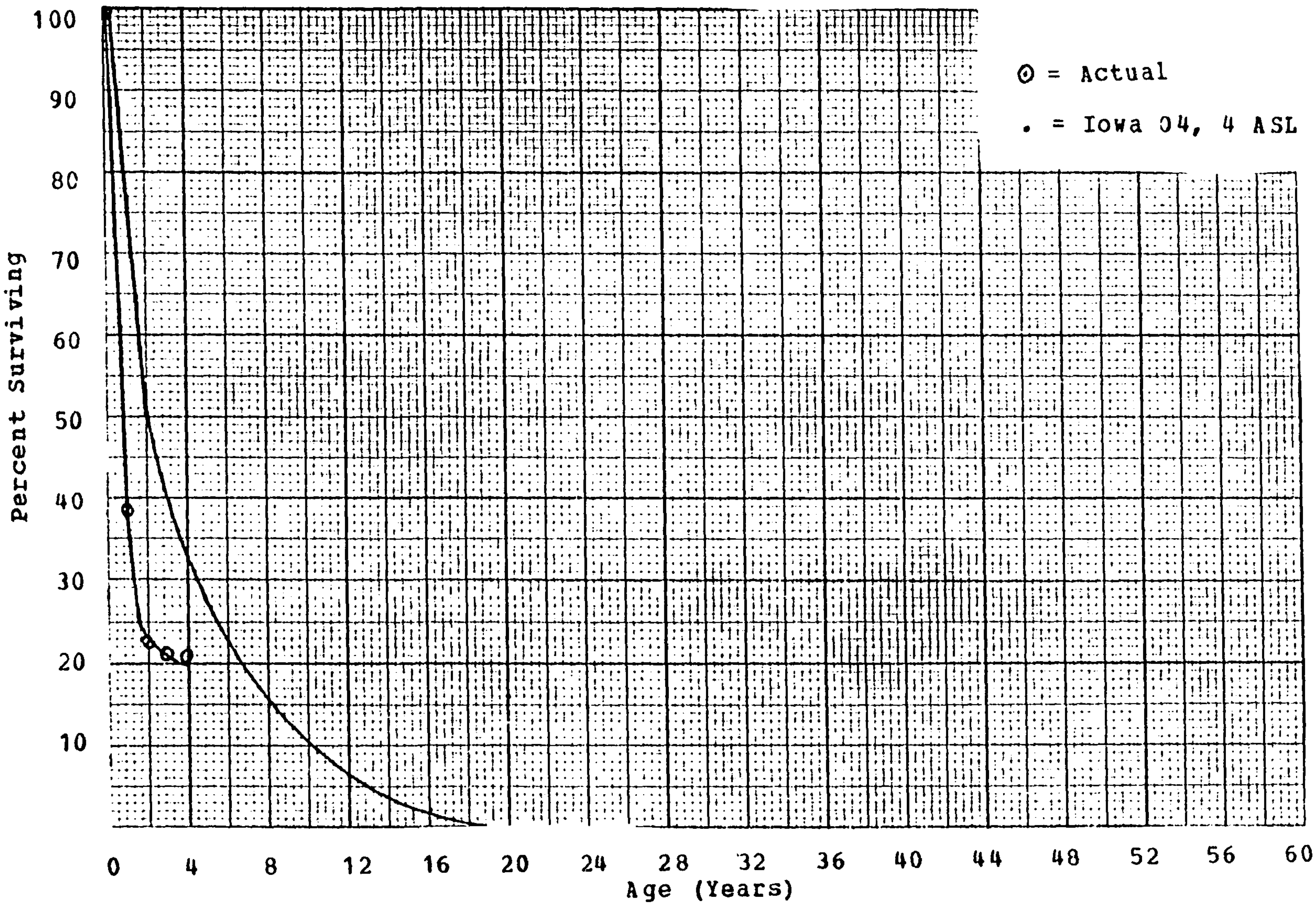

Figure 54. Utility Coal and Ash Handlers, original group, 1970-72

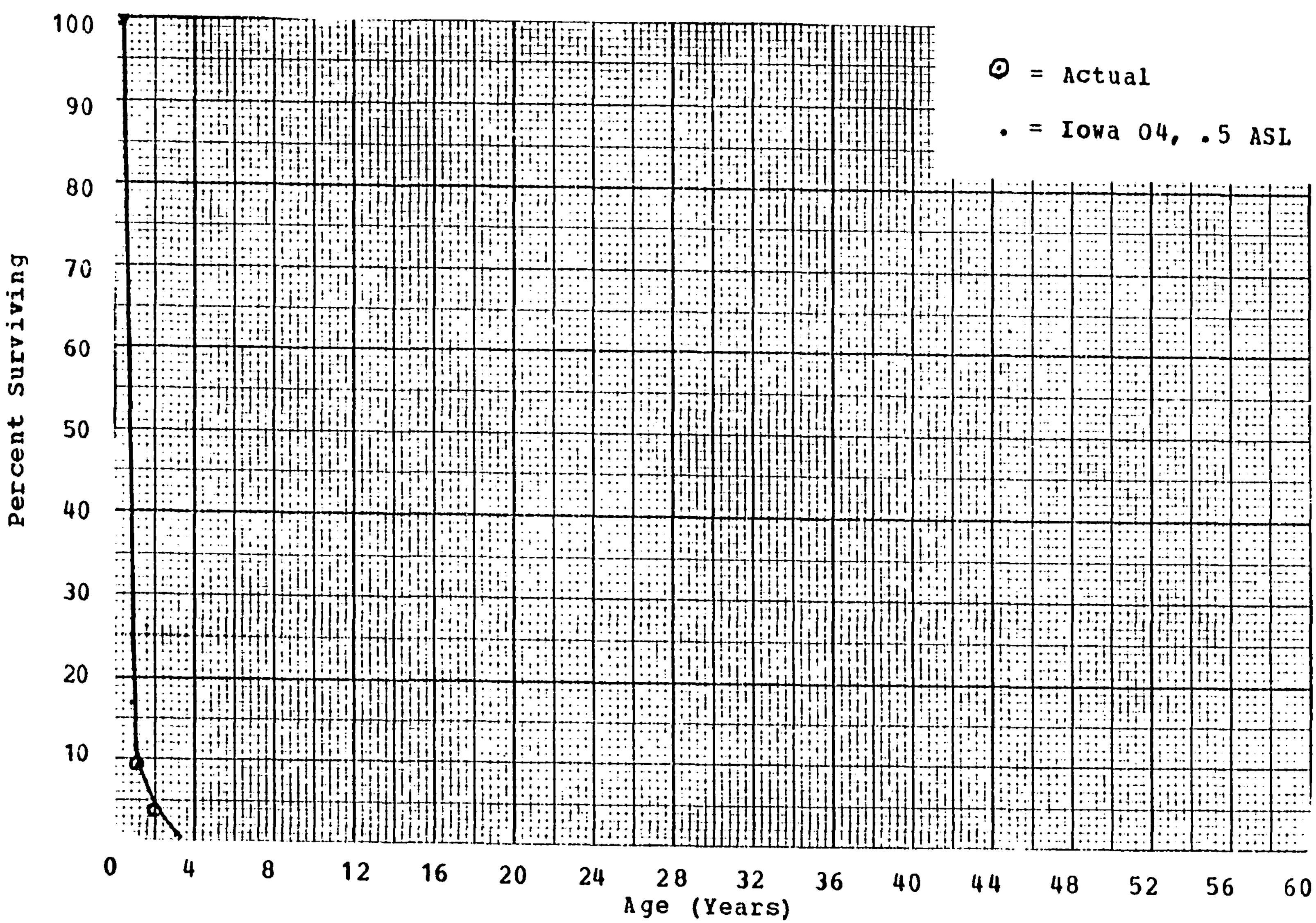

Figure 55. Utility Ironworkers, original group, 1969-71

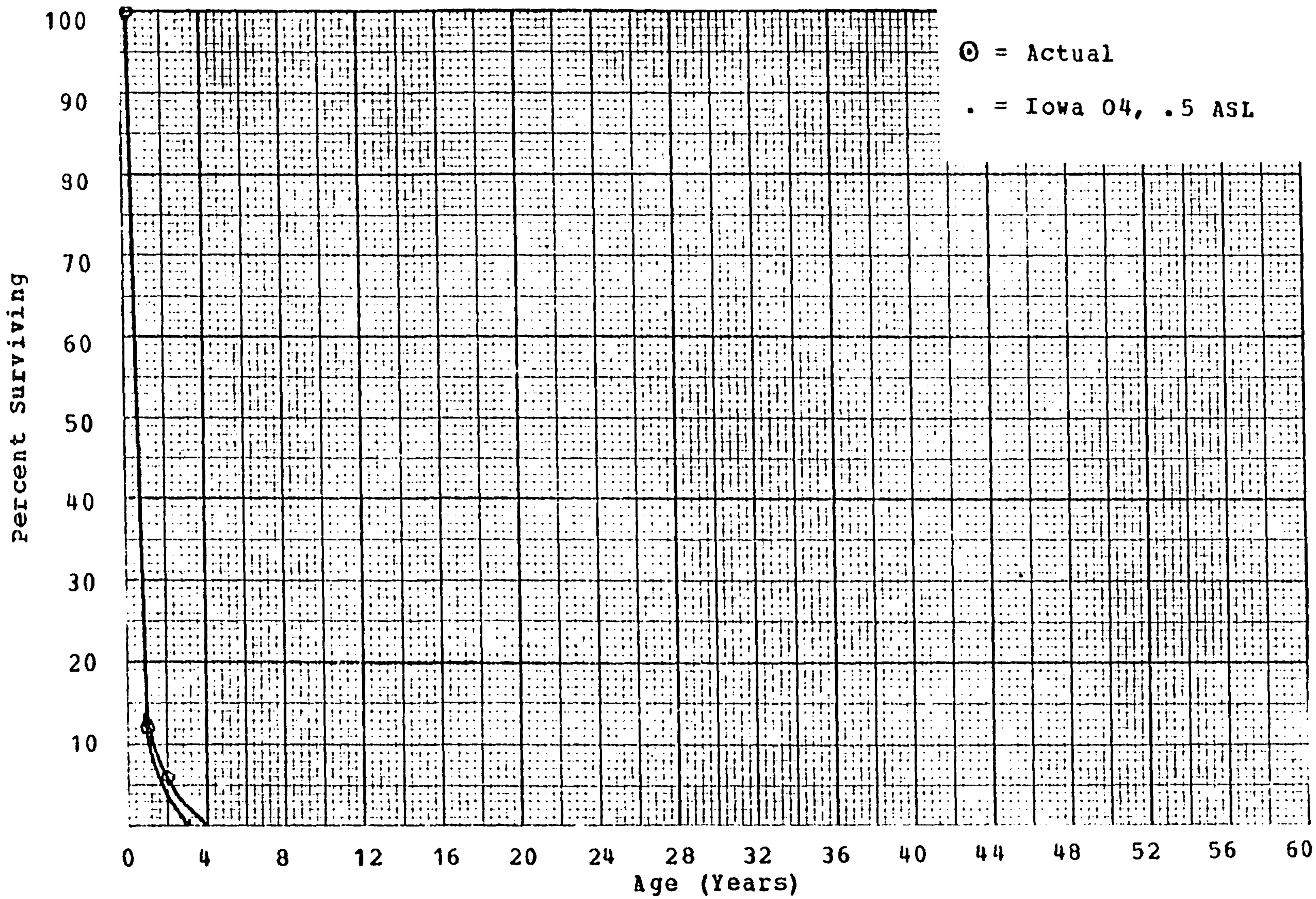

Figure 56. Utility Ironworkers, original group, 1970-72

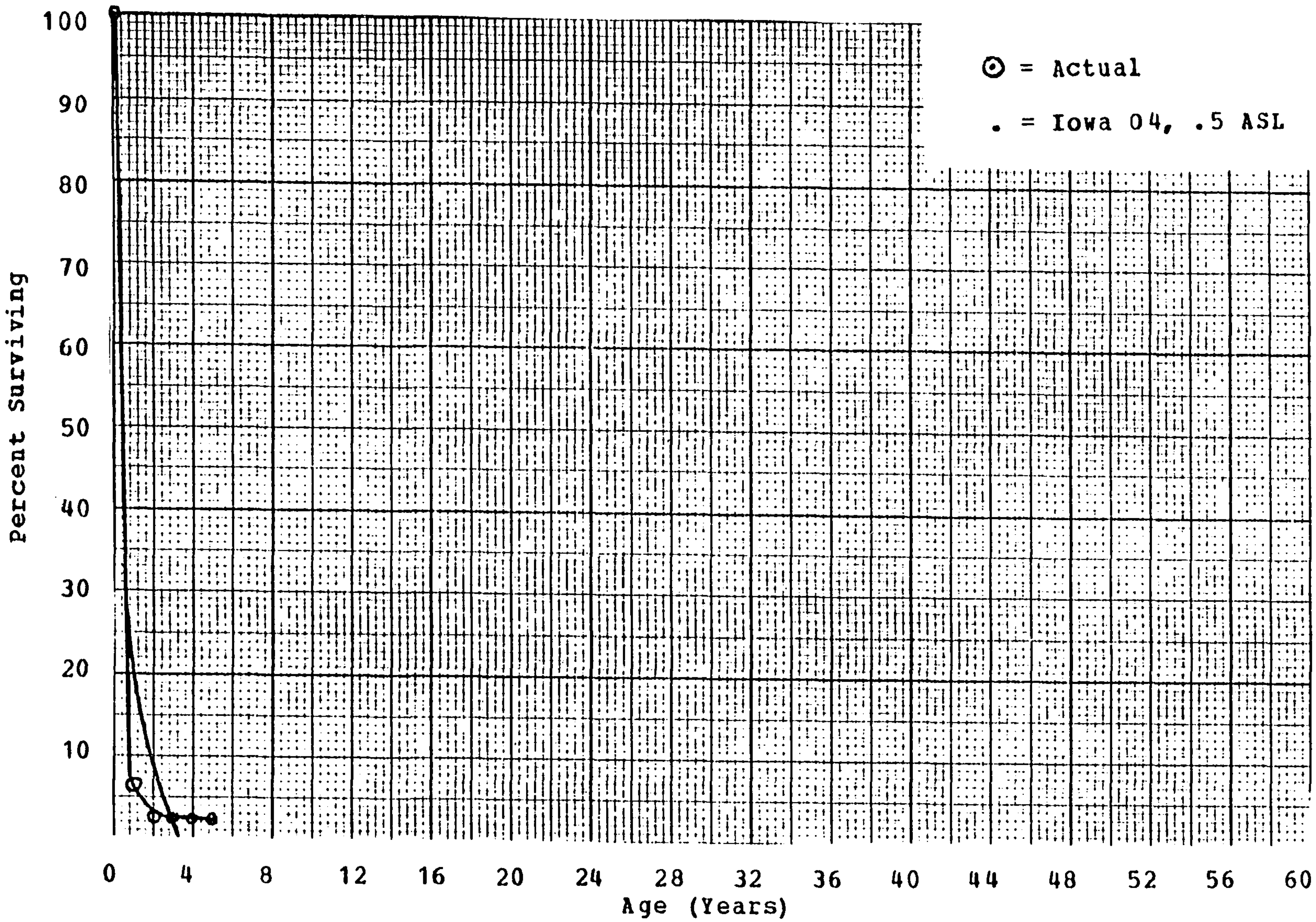

Figure 57. Utility Laborers, original group, 1969-71

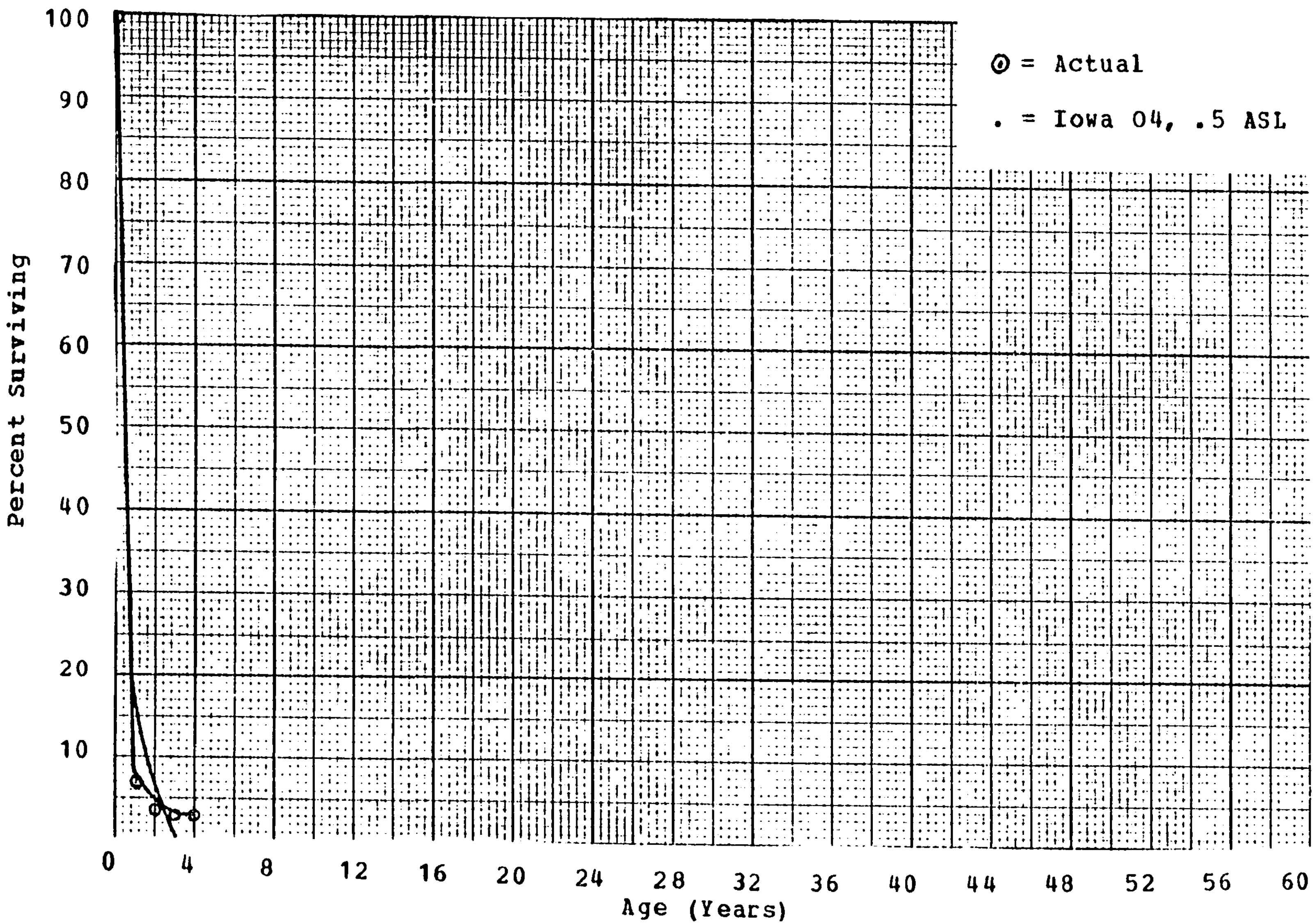

Figure 58. Utility Laborers, original group, 1970-72

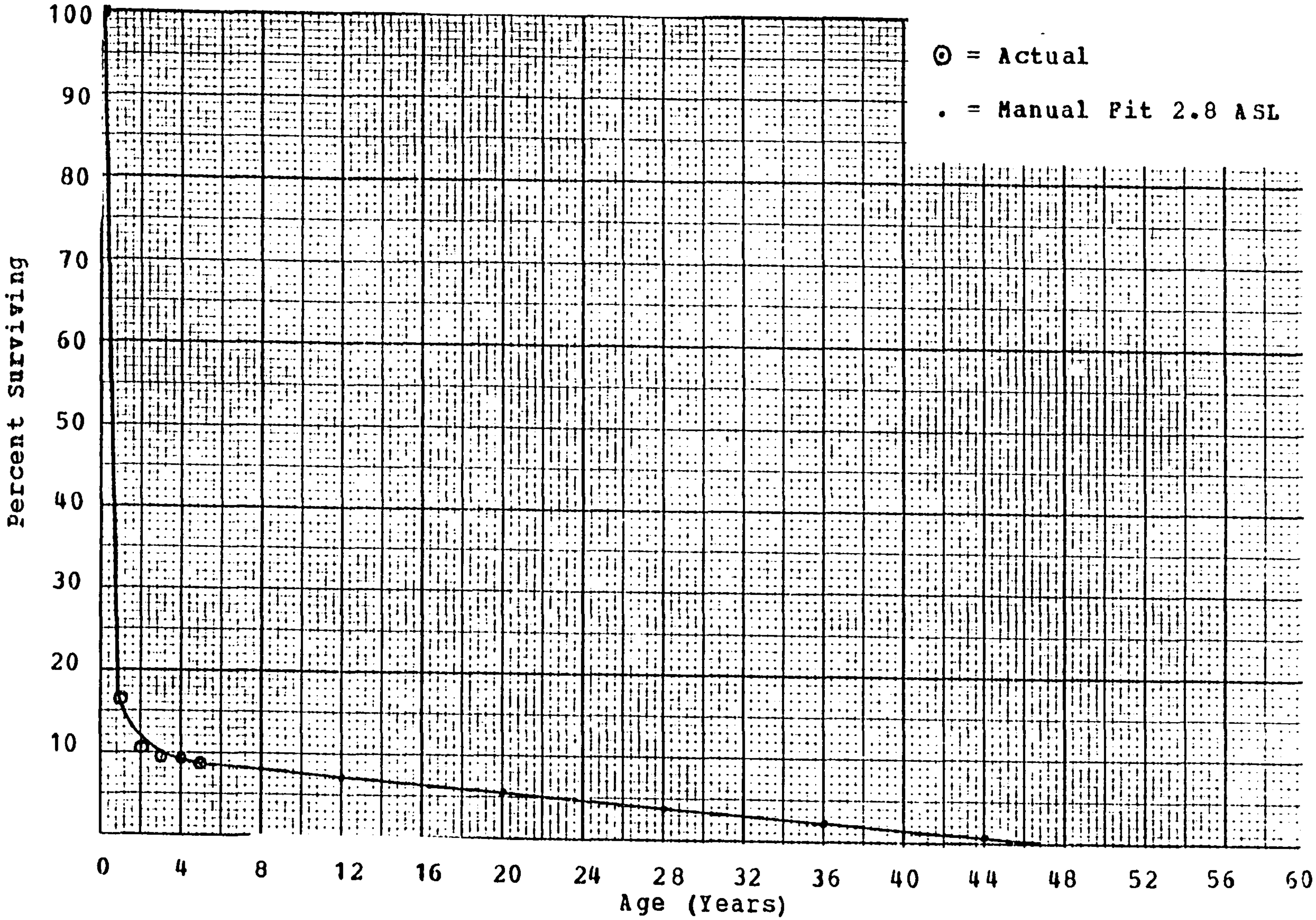

Figure 59. Utility Union Employees, original group, 1969-71

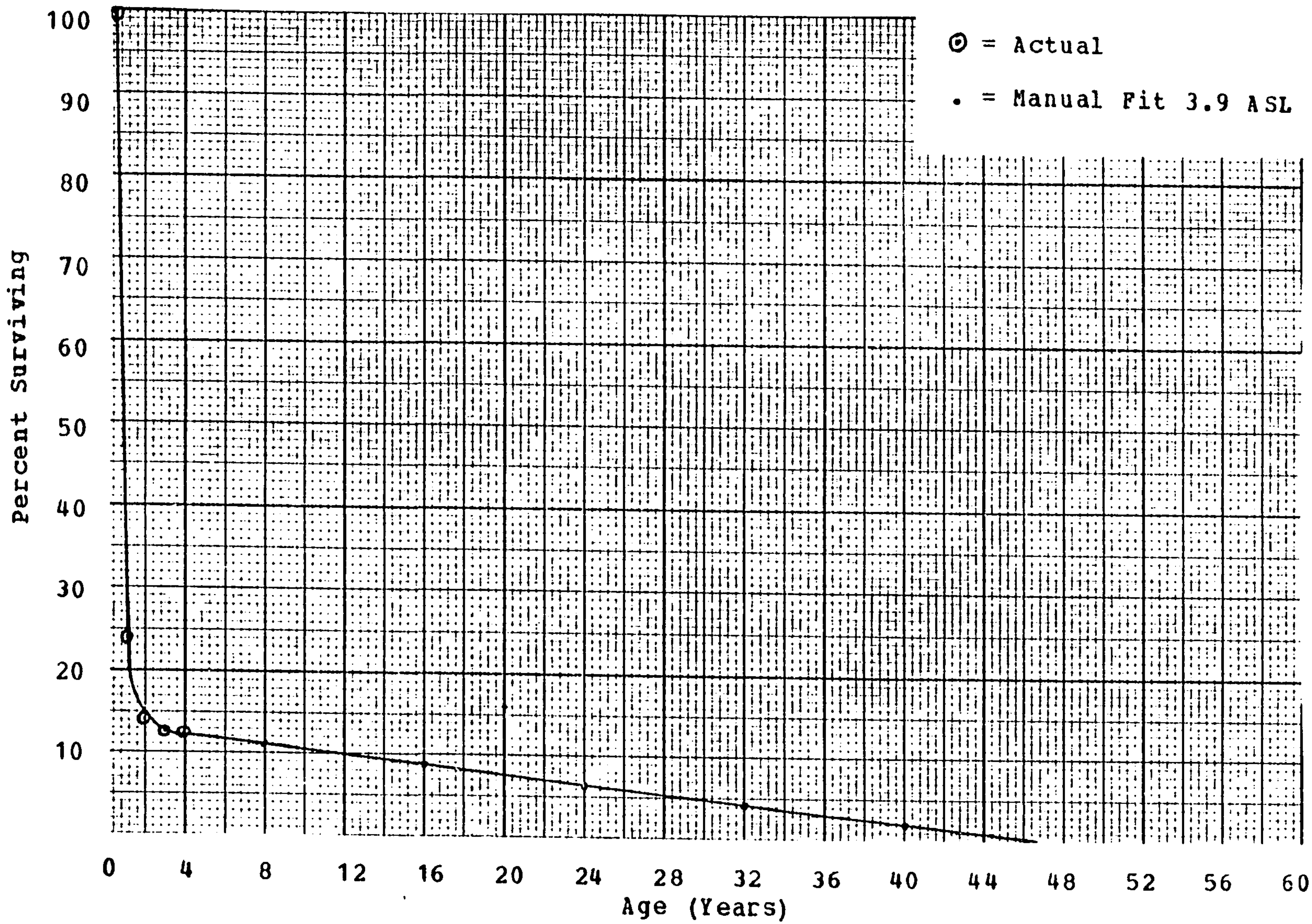

Figure 60. Utility Union Employees, original group, 1970-72

APPENDIX C: HISTORICAL ARRANGEMENT OF DATA, TREND ANALYSIS, SIMULATED PLANT RECORD TABLES, AND CHI-SQUARE DATA

Table 9. Historical arrangement of Manufacturing Marketing Employees

HISTORICAL ARRANGEMENT OF MORTALITY DATA PAGE 1 OF 4

MFG 1

YEAR	PLANT IN SERVICE JANUARY 1	RETIRED DURING YEAR	PLANT ADD. RETIRED FROM THESE ADDITIONS	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM 0.5	1.5	2.5	3.5	4.5	5.5	6.5
1941	C.		1.	0.	0.	0.	0.	0.	0.	0.
		0.	0.	0.	0.	0.	0.	0.	0.	0.
1942	1.		0.	1.	0.	0.	0.	0.	0.	0.
		0.	0.	0.	0.	0.	0.	0.	0.	0.
1943	1.		0.	0.	1.	0.	0.	0.	0.	0.
		0.	0.	0.	0.	0.	0.	0.	0.	0.
1944	1.		1.	0.	0.	1.	0.	0.	0.	0.
		0.	0.	0.	0.	0.	0.	0.	0.	0.
1945	2.		2.	1.	0.	0.	1.	0.	0.	0.
		0.	0.	0.	0.	0.	0.	0.	0.	0.
1946	4.		6.	2.	1.	0.	0.	1.	0.	0.
		0.	0.	0.	0.	0.	0.	0.	0.	0.
1947	10.		12.	6.	2.	1.	0.	0.	1.	0.
		1.	1.	0.	0.	0.	0.	0.	0.	0.
1948	21.		14.	11.	6.	2.	1.	0.	0.	1.
		3.	3.	0.	0.	0.	0.	0.	0.	0.
1949	32.		16.	11.	11.	6.	2.	1.	0.	0.
		1.	1.	0.	0.	0.	0.	0.	0.	0.
1950	47.		7.	15.	11.	11.	6.	2.	1.	0.
		5.	0.	3.	1.	1.	0.	0.	0.	0.
1951	49.		7.	7.	12.	10.	10.	6.	2.	1.
		3.	1.	0.	1.	1.	0.	0.	0.	0.
1952	53.		6.	5.	7.	11.	9.	10.	6.	2.
		1.	0.	1.	0.	0.	0.	0.	0.	0.
1953	58.		13.	6.	5.	7.	11.	9.	10.	6.
		1.	0.	0.	0.	0.	0.	0.	1.	0.
1954	70.		14.	13.	6.	5.	7.	11.	9.	9.
		3.	1.	1.	0.	0.	1.	0.	0.	0.
1955	81.		5.	13.	12.	6.	5.	6.	11.	9.
		5.	0.	1.	3.	1.	0.	0.	0.	0.
1956	81.		13.	5.	12.	9.	5.	5.	6.	11.
		4.	0.	0.	0.	1.	0.	1.	0.	0.
1957	90.		6.	13.	5.	12.	8.	5.	4.	6.
		6.	0.	2.	0.	1.	1.	0.	1.	0.
1958	90.		6.	6.	11.	5.	11.	7.	5.	3.
		4.	0.	0.	2.	0.	1.	0.	0.	0.

Table 9. (Continued)

1959	92.		13.	6.	6.	9.	5.	10.	7.	5.
		9.	0.	1.	2.	1.	0.	0.	0.	1.
1960	96.		14.	13.	5.	4.	9.	5.	10.	7.
		4.	0.	0.	0.	0.	1.	0.	0.	1.
1961	106.		13.	14.	13.	5.	4.	7.	5.	10.
		8.	0.	0.	4.	0.	0.	2.	0.	1.
1962	111.		11.	13.	14.	9.	5.	4.	5.	5.
		7.	1.	1.	0.	2.	0.	0.	1.	0.
1963	115.		17.	10.	12.	14.	7.	5.	4.	4.
		6.	1.	1.	2.	0.	1.	0.	0.	0.
1964	126.		13.	16.	9.	10.	14.	6.	5.	4.
		11.	0.	0.	1.	0.	3.	2.	1.	0.
1965	128.		24.	13.	16.	9.	10.	11.	4.	4.
		11.	0.	2.	3.	3.	0.	0.	0.	2.
1966	141.		21.	24.	11.	13.	5.	10.	11.	4.
		14.	1.	2.	3.	1.	0.	2.	2.	0.
1967	148.		27.	20.	22.	8.	12.	5.	8.	9.
		6.	0.	0.	2.	3.	0.	1.	0.	0.
1968	169.		24.	27.	20.	20.	5.	12.	4.	8.
		19.	2.	7.	2.	4.	1.	1.	0.	0.
1969	174.		40.	22.	20.	13.	16.	4.	11.	4.
		17.	1.	1.	4.	4.	6.	0.	0.	0.
1970	197.		35.	39.	21.	16.	14.	10.	4.	11.
		20.	1.	4.	8.	1.	1.	1.	0.	1.
1971	212.		23.	34.	35.	13.	15.	13.	9.	4.
		18.	0.	3.	1.	3.	4.	1.	2.	1.
1972	217.		35.	23.	31.	34.	10.	11.	12.	7.
		20.	1.	6.	2.	4.	0.	1.	2.	0.
1973	232.		33.	34.	17.	29.	30.	10.	10.	10.
		17.	2.	2.	2.	3.	4.	0.	1.	0.
TOTAL EXPOSURES			472.	424.	354.	296.	236.	186.	164.	144.
TOTAL RETIREMENTS			17.	38.	43.	34.	24.	12.	11.	7.

Table 9. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA
MFG1

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5
1941	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1942	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1943	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1944	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1945	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1946	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1947	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1948	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1949	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	[illegible]	0.	0.	0.	0.	0.	0.	0.	0.	0.
1950	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1951	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1952	1.	0.	0.	1.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1953	2.	1.	0.	0.	1.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1954	6.	2.	1.	0.	0.	1.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1955	9.	6.	2.	1.	0.	0.	1.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1956	9.	9.	6.	2.	1.	0.	0.	1.	0.	0.
	2.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1957	11.	7.	9.	6.	2.	1.	0.	0.	1.	0.
	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.
1958	8.	11.	8.	9.	6.	2.	1.	0.	0.	1.
	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.

Table 9. (Continued)

1959	5.	6.	10.	6.	9.	6.	2.	1.	0.	0.
	0.	1.	1.	1.	1.	0.	0.	0.	0.	0.
1960	4.	3.	5.	9.	5.	8.	6.	2.	1.	0.
	1.	0.	1.	0.	0.	0.	0.	0.	0.	0.
1961	6.	3.	3.	4.	9.	5.	8.	6.	2.	1.
	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.
1962	9.	6.	3.	3.	4.	8.	5.	8.	6.	2.
	1.	0.	1.	0.	0.	0.	0.	0.	0.	0.
1963	5.	8.	6.	2.	3.	4.	8.	5.	8.	6.
	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.
1964	4.	5.	8.	6.	2.	2.	4.	8.	5.	8.
	1.	0.	1.	0.	0.	0.	0.	2.	0.	0.
1965	4.	3.	5.	7.	6.	2.	2.	4.	6.	5.
	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.
1966	2.	4.	3.	4.	7.	6.	2.	2.	4.	6.
	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.
1967	4.	2.	4.	3.	3.	7.	6.	2.	2.	4.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1968	9.	4.	2.	4.	3.	3.	7.	6.	2.	2.
	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1969	8.	3.	4.	2.	4.	5.	3.	7.	6.	2.
	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	4.	7.	9.	4.	2.	4.	3.	3.	7.	6.
	0.	1.	1.	0.	0.	1.	0.	0.	0.	0.
1971	10.	4.	6.	7.	4.	2.	3.	3.	3.	7.
	1.	1.	0.	0.	0.	0.	0.	0.	0.	1.
1972	3.	9.	3.	6.	7.	4.	2.	3.	3.	3.
	0.	1.	1.	0.	0.	0.	0.	0.	1.	0.
1973	7.	3.	8.	2.	6.	7.	4.	2.	3.	2.
	1.	0.	0.	0.	1.	0.	1.	0.	0.	0.
TOTAL EXP.	127.	112.	103.	88.	84.	75.	67.	63.	59.	55.
TOTAL RET.	9.	6.	7.	2.	4.	1.	1.	2.	1.	1.

Table 9. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA PAGE 3 OF 4

HFS1

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	17.5	18.5	19.5	20.5	21.5	22.5	23.5	24.5	25.5	26.5
1941	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1942	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1943	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1944	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1945	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1946	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1947	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1948	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1949	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1950	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1951	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1952	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1953	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1954	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1955	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1956	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1957	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

Table 9. (Continued)

1958	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1959	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1960	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1961	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1962	1.	0.	0.	1.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1963	2.	1.	0.	0.	1.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1964	6.	2.	1.	0.	0.	1.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1965	8.	6.	2.	1.	0.	0.	1.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1966	5.	8.	6.	2.	1.	0.	0.	1.	0.	0.
	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.
1967	6.	4.	8.	6.	2.	0.	0.	0.	1.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1968	4.	6.	4.	8.	5.	2.	0.	0.	0.	1.
	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.
1969	2.	4.	6.	3.	8.	6.	2.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	2.	2.	4.	6.	3.	8.	6.	2.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	6.	2.	2.	4.	6.	3.	8.	6.	2.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	6.	6.	2.	2.	4.	6.	3.	8.	6.	2.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.
1973	3.	6.	6.	2.	2.	4.	6.	3.	8.	6.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	52.	48.	42.	35.	33.	30.	26.	20.	17.	9.
TOTAL RET.	0.	0.	1.	0.	1.	0.	0.	0.	0.	1.

Table 9. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA
MFG1

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	27.5	28.5	29.5	30.5	31.5
1941	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1942	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1943	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1944	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1945	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1946	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1947	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1948	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1949	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1950	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1951	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1952	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1953	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1954	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1955	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1956	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1957	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.

Table 9. (Continued)

1958	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1959	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1960	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1961	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1962	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1963	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1964	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1965	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1966	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1967	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1968	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1969	1.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
1970	0.	1.	0.	0.	0.
	0.	0.	0.	0.	0.
1971	0.	0.	1.	0.	0.
	0.	0.	0.	0.	0.
1972	0.	0.	0.	1.	0.
	0.	0.	0.	0.	0.
1973	1.	0.	0.	0.	1.
	0.	0.	0.	0.	0.
TOTAL EXP.	2.	1.	1.	1.	1.
TOTAL RET.	0.	0.	0.	0.	0.

Table 10. Actuarial trend analysis of Manufacturing Marketing Employees

IOWA STATE UNIVERSITY
ACTUARIAL TREND ANALYSIS
MF31

RETIREMENT BAND	INDICATED AVERAGE LIFE FIRST DEGREE	SECOND DEGREE	THIRD DEGREE	ACTUAL RETIREMENTS FITTED	INDICATED RETIREMENTS FITTED FIRST DEGREE	SECOND DEGREE	THIRD DEGREE
1941-1943	199.0	199.0	199.0	0.	0.	0.	0.
1942-1944	199.0	199.0	199.0	0.	0.	0.	0.
1943-1945	199.0	199.0	199.0	0.	0.	0.	0.
1944-1946	199.0	199.0	199.0	0.	0.	0.	0.
1945-1947	199.0	199.0	199.0	0.	0.	0.	0.
1946-1948	199.0	199.0	199.0	0.	0.	0.	0.
1947-1949	199.0	199.0	[illegible]	0.	0.	0.	0.
1948-1950	155.9	13.3	155.9	5.	5.	5.	5.
1949-1951	146.2	14.5	[illegible]	7.	7.	7.	7.
1950-1952	136.[illegible]	[illegible]	132.1	8.	9.	8.	8.
1951-1953	153.0	15.5	[illegible]	4.	4.	4.	4.
1952-1954	163.4	17.4	[illegible]	4.	4.	4.	4.
1953-1955	135.7	13.1	[illegible]	8.	8.	8.	8.
1954-1956	120.7	[illegible]	[illegible]	11.	11.	11.	11.
1955-1957	[illegible]	[illegible]	[illegible]	15.	15.	15.	15.
1956-1958	100.7	[illegible]	[illegible]	14.	14.	14.	14.
1957-1959	75.2	[illegible]	[illegible]	19.	19.	19.	19.
1958-1960	74.5	[illegible]	[illegible]	17.	17.	17.	17.
1959-1961	63.5	[illegible]	[illegible]	21.	21.	21.	21.
1960-1962	[illegible]	76.2	[illegible]	18.	18.	18.	18.
1961-1963	[illegible]	[illegible]	[illegible]	19.	19.	19.	19.
1962-1964	[illegible]	[illegible]	[illegible]	22.	22.	22.	22.
1963-1965	56.7	[illegible]	[illegible]	27.	27.	27.	27.
1964-1966	60.2	35.8	[illegible]	35.	35.	35.	35.
1965-1967	55.6	[illegible]	[illegible]	30.	30.	30.	30.
1966-1968	50.5	[illegible]	[illegible]	36.	36.	36.	36.
1967-1969	50.0	[illegible]	[illegible]	39.	[illegible]	39.	39.
1968-1970	37.0	[illegible]	[illegible]	52.	[illegible]	52.	52.
1969-1971	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	54.	54.
1970-1972	38.2	13.8	[illegible]	56.	56.	56.	56.
1971-1973	44.3	20.7	[illegible]	52.	52.	52.	52.
1941-1973	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	207.
1942-1973	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1943-1973	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	207.	[illegible]
1944-1973	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1945-[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 10. (Continued)

1946-1973	[illegible]	[illegible]	14.9	[illegible]	[illegible]	[illegible]	207.
1947-1973	63.1	17.3	14.8	207.	209.	207.	207.
1948-1973	62.3	17.1	14.8	207.	209.	207.	207.
1949-1973	62.2	16.7	14.8	207.	209.	207.	207.
1950-1973	61.3	16.3	14.7	207.	209.	207.	207.
1951-1973	61.3	15.4	14.5	202.	204.	202.	202.
1952-1973	60.3	16.2	14.3	200.	202.	200.	200.
1953-1973	58.9	16.0	13.9	199.	201.	199.	199.
1954-1973	57.4	15.7	13.6	198.	201.	198.	198.
1955-1973	55.6	15.5	13.2	196.	199.	196.	196.
1956-1973	54.4	15.5	12.9	191.	193.	191.	191.
1957-1973	53.2	15.3	12.7	187.	189.	187.	187.
1958-1973	52.2	15.0	12.5	181.	183.	181.	181.
1959-1973	50.5	15.8	12.2	177.	179.	177.	177.
1960-1973	51.1	15.7	12.3	168.	170.	168.	168.
1961-1973	49.3	15.0	12.1	164.	166.	164.	164.
1962-1973	48.7	14.9	12.1	156.	158.	156.	156.
1963-1973	47.1	[illegible]	11.9	150.	152.	150.	150.
1964-1973	44.6	14.4	11.4	145.	147.	145.	145.
1965-1973	45.1	13.5	11.6	134.	136.	134.	134.
1966-1973	[illegible]	[illegible]	11.8	123.	125.	123.	123.
1967-1973	[illegible]	[illegible]	12.1	[illegible]	[illegible]	[illegible]	110.
1968-1973	[illegible]	[illegible]	11.9	104.	106.	[illegible]	104.
1969-1973	42.5	[illegible]	11.5	87.	[illegible]	87.	87.
1970-1973	42.7	13.4	11.7	71.	71.	71.	71.
1971-1973	[illegible]	[illegible]	12.0	52.	52.	52.	52.
1972-1973	45.2	14.7	13.8	34.	34.	34.	34.
1973-1973	[illegible]	56.3	15.1	15.	15.	15.	15.

Table 11. Actuarial trend analysis of Office, Career College Graduates

IOWA STATE UNIVERSITY
ACTUARIAL TREND ANALYSIS
OFFICE1

RETIREMENT BAND	INDICATED AVERAGE LIFE FIRST DEGREE	SECOND DEGREE	THIRD DEGREE	ACTUAL RETIREMENTS FITTED	INDICATED RETIREMENTS FITTED FIRST DEGREE	SECOND DEGREE	THIRD DEGREE
1961-1963	127.4	130.6	9.8	10.	10.	10.	10.
1962-1964	91.6	2.9	2.9	21.	21.	21.	21.
1963-1965	85.4	2.9	49.6	37.	37.	37.	37.
1964-1966	84.6	3.6	54.9	52.	52.	52.	52.
1965-1967	82.8	4.3	66.4	56.	56.	56.	56.
1966-1968	77.5	5.2	12.1	59.	59.	59.	59.
1967-1969	77.7	9.4	5.9	55.	55.	55.	55.
1968-1970	56.0	4.8	45.9	74.	74.	74.	74.
1969-1971	65.5	6.1	59.8	65.	65.	65.	65.
1961-1971	70.5	6.3	62.7	165.	165.	165.	165.
1962-1971	70.5	6.3	62.7	165.	165.	165.	165.
1963-1971	70.3	6.2	62.0	161.	161.	161.	161.
1964-1971	69.1	6.1	60.4	155.	155.	155.	155.
1965-1971	68.4	6.0	59.7	144.	144.	144.	144.
1966-1971	69.7	6.2	62.0	124.	124.	124.	124.
1967-1971	68.9	6.8	63.6	103.	103.	103.	103.
1968-1971	64.5	6.0	58.6	88.	88.	88.	88.
1969-1971	65.5	6.1	59.8	65.	65.	65.	65.
1970-1971	58.4	5.8	52.7	48.	48.	48.	48.
1971-1971	91.7	10.5	7.7	14.	15.	14.	14.

Table 12. Historical arrangement of Utility Meter Readers

HISTORICAL ARRANGEMENT OF MORTALITY DATA

UTIL-MR

YEAR	PLANT IN SERVICE JANUARY 1	RETIRED DURING YEAR	PLANT ADD. RETIRED FROM THESE ADDITIONS	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM 0.5	1.5	2.5	3.5	4.5	5.5	6.5
1969	137.		25.	3.	3.	3.	2.	7.	6.	7.
		22.	16.	0.	1.	0.	0.	0.	0.	1.
1970	140.		28.	9.	3.	2.	3.	2.	7.	6.
		28.	20.	5.	0.	0.	0.	1.	0.	1.
1971	140.		25.	8.	4.	3.	2.	3.	1.	7.
		19.	17.	0.	0.	0.	0.	1.	0.	0.
1972	146.		71.	8.	8.	4.	3.	2.	2.	1.
		29.	25.	0.	0.	0.	1.	0.	0.	0.
1973	188.		45.	46.	8.	8.	4.	2.	2.	2.
		41.	25.	13.	0.	0.	1.	0.	0.	0.
TOTAL EXPOSURES			194.	74.	26.	20.	14.	16.	15.	23.
TOTAL RETIREMENTS			105.	18.	1.	0.	2.	2.	0.	2.

YEAR	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM 7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5
1969	2.	0.	8.	4.	7.	9.	1.	4.	11.	8.
	0.	0.	0.	0.	0.	0.	0.	1.	1.	0.
1970	6.	2.	0.	8.	4.	7.	9.	1.	3.	10.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	5.	6.	2.	0.	8.	4.	7.	9.	1.	3.
	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.
1972	7.	5.	6.	2.	0.	7.	4.	7.	9.	1.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1973	1.	7.	5.	6.	2.	0.	7.	4.	7.	9.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	21.	20.	21.	20.	21.	27.	26.	25.	31.	31.
TOTAL RET.	0.	0.	0.	0.	1.	0.	0.	1.	1.	0.

Table 12. (continued)

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	17.5	18.5	19.5	20.5	21.5	22.5	23.5	24.5	25.5	26.5
1969	8.	6.	3.	7.	9.	4.	4.	2.	3.	1.
	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.
1970	8.	8.	6.	3.	6.	9.	4.	4.	2.	3.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.
1971	10.	8.	8.	6.	3.	6.	9.	4.	4.	2.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	3.	10.	8.	8.	6.	3.	6.	9.	4.	4.
	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.
1973	1.	3.	10.	8.	8.	6.	3.	6.	8.	4.
	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.
TOTAL EXP.	30.	35.	35.	32.	32.	28.	26.	25.	21.	14.
TOTAL RET.	0.	0.	0.	1.	0.	0.	0.	2.	0.	1.

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	27.5	28.5	29.5	30.5	31.5	32.5	33.5	34.5	35.5	36.5
1969	1.	0.	0.	0.	2.	1.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	1.	1.	0.	0.	0.	2.	1.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	2.	1.	1.	0.	0.	0.	2.	1.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	2.	2.	1.	1.	0.	0.	0.	2.	1.	0.
	1.	0.	1.	0.	0.	0.	0.	0.	0.	0.
1973	4.	1.	2.	0.	1.	0.	0.	0.	2.	1.
	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.
TOTAL EXP.	10.	5.	4.	1.	3.	3.	3.	3.	3.	1.
TOTAL RET.	1.	0.	1.	0.	0.	0.	0.	0.	1.	0.

Table 12. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA
UTIL-MR

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	37.5	38.5
1969	0.	1.
	0.	1.
1970	0.	0.
	0.	0.
1971	0.	0.
	0.	0.
1972	0.	0.
	0.	0.
1973	0.	0.
	0.	0.
TOTAL EXP.	0.	1.
TOTAL RET.	0.	1.

Table 13. Actuarial trend analysis of Utility Meter Readers

IOWA STATE UNIVERSITY
ACTUARIAL TREND ANALYSIS
UTIL-MR

RETIREMENT BAND	INDICATED AVERAGE LIFE FIRST DEGREE	SECOND DEGREE	THIRD DEGREE	ACTUAL RETIREMENTS FITTED	INDICATED RETIREMENTS FITTED FIRST DEGREE	SECOND DEGREE	THIRD DEGREE
1969-1971	64.4	13.0	13.0	16.	16.	16.	16.
1970-1972	75.4	16.6	43.0	14.	14.	14.	14.
1971-1973	54.4	14.7	19.2	22.	22.	23.	23.
1969-1973	51.5	12.7	13.1	36.	36.	37.	37.
1970-1973	48.7	12.6	23.6	30.	31.	31.	32.
1971-1973	54.4	14.7	19.2	22.	22.	23.	23.
1972-1973	39.7	12.9	16.2	20.	20.	21.	21.
1973-1973	24.2	10.1	11.3	16.	16.	17.	17.

Table 14. Historical arrangement of Utility Mechanics

HISTORICAL ARRANGEMENT OF MORTALITY DATA — PAGE 1 OF 5

UTIL-MEC

YEAR	PLANT IN SERVICE JANUARY 1	RETIRED DURING YEAR	PLANT ADD. RETIRED FROM THESE ADDITIONS	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM 0.5	1.5	2.5	3.5	4.5	5.5	6.5
1969	84.		30.	9.	6.	3.	2.	7.	2.	4.
		22.	19.	1.	0.	1.	0.	1.	0.	0.
1970	92.		23.	11.	8.	6.	2.	2.	6.	2.
		30.	17.	7.	1.	2.	0.	0.	0.	1.
1971	85.		17.	6.	4.	7.	4.	2.	2.	6.
		11.	10.	0.	0.	0.	0.	0.	0.	0.
1972	91.		17.	7.	6.	4.	7.	4.	2.	2.
		13.	12.	0.	0.	0.	0.	0.	0.	0.
1973	95.		16.	5.	7.	6.	4.	7.	4.	2.
		14.	9.	4.	0.	0.	0.	0.	0.	0.
TOTAL EXPOSURES			103.	38.	31.	26.	19.	22.	16.	16.
TOTAL RETIREMENTS			67.	12.	1.	3.	0.	1.	0.	1.

YEAR	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM 7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5
1969	2.	1.	3.	3.	8.	4.	3.	6.	2.	5.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	4.	2.	1.	3.	3.	8.	4.	3.	6.	2.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	1.	4.	2.	1.	3.	3.	8.	4.	3.	6.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	6.	1.	4.	2.	1.	3.	3.	8.	4.	3.
	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.
1973	2.	6.	1.	4.	1.	1.	3.	3.	8.	4.
	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.
TOTAL EXP.	15.	14.	11.	13.	16.	19.	21.	24.	23.	20.
TOTAL RET.	0.	0.	0.	1.	0.	0.	0.	0.	1.	0.

Table 14. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA
UTIL-MEC

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	17.5	18.5	19.5	20.5	21.5	22.5	23.5	24.5	25.5	26.5
1969	2.	2.	1.	5.	0.	1.	0.	0.	0.	2.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	5.	2.	2.	1.	5.	0.	1.	0.	0.	0.
	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.
1971	2.	5.	2.	2.	1.	4.	0.	1.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	6.	2.	5.	2.	2.	1.	4.	0.	1.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1973	3.	6.	2.	5.	2.	2.	1.	4.	0.	1.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	18.	17.	12.	15.	10.	8.	6.	5.	1.	3.
TOTAL RET.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	27.5	28.5	29.5	30.5	31.5	32.5	33.5	34.5	35.5	36.5
1969	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	2.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	0.	2.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.
1972	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1973	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	2.	2.	1.	1.	0.	0.	0.	0.	0.	0.
TOTAL RET.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.

Table 14. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA

UTIL-MEC

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	37.5	38.5	39.5	40.5	41.5	42.5	43.5	44.5
1969	0.	0.	0.	0.	0.	0.	1.	0.
	0.	0.	0.	0.	0.	0.	0.	0.
1970	0.	0.	0.	0.	0.	0.	0.	1.
	0.	0.	0.	0.	0.	0.	0.	1.
1971	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.
1972	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.
1973	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	0.	0.	0.	0.	0.	0.	1.	1.
TOTAL RET.	0.	0.	0.	0.	0.	0.	0.	1.

Table 15. Actuarial trend analysis of Utility Mechanics

IOWA STATE UNIVERSITY
ACTUARIAL TREND ANALYSIS
UTIL-MEC

RETIREMENT BAND	INDICATED AVERAGE LIFE FIRST DEGREE	SECOND DEGREE	THIRD DEGREE	ACTUAL RETIREMENTS FITTED	INDICATED RETIREMENTS FITTED FIRST DEGREE	SECOND DEGREE	THIRD DEGREE
1969-1971	46.8	11.1	10.5	17.	17.	17.	18.
1970-1972	53.2	12.4	12.4	15.	15.	16.	16.
1971-1973	108.6	21.1	33.9	7.	7.	7.	7.
1969-1973	63.0	13.5	13.1	23.	23.	24.	25.
1970-1973	57.2	12.1	11.9	20.	20.	22.	22.
1971-1973	108.6	21.1	33.9	7.	7.	7.	7.
1972-1973	92.4	20.0	92.5	6.	6.	6.	7.
1973-1973	52.6	11.2	39.9	5.	5.	6.	6.

Table 16. Historical arrangement of Utility Coal and Ash Handlers

HISTORICAL ARRANGEMENT OF MORTALITY DATA PAGE 1 OF 5

UTIL-CAH

YEAR	PLANT IN SERVICE JANUARY 1	RETIRED DURING YEAR	PLANT ADD. RETIRED FROM THESE ADDITIONS	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM 0.5	1.5	2.5	3.5	4.5	5.5	6.5
1969	34.		89.	17.	0.	6.	1.	0.	1.	1.
		83.	69.	12.	0.	0.	0.	0.	1.	1.
1970	40.		94.	20.	5.	0.	6.	1.	0.	0.
		74.	63.	11.	0.	0.	0.	0.	0.	0.
1971	60.		72.	31.	9.	5.	0.	6.	1.	0.
		57.	46.	10.	0.	0.	0.	0.	0.	0.
1972	75.		78.	26.	21.	9.	5.	0.	6.	1.
		59.	41.	15.	2.	0.	0.	0.	1.	0.
1973	94.		66.	37.	11.	19.	9.	5.	0.	5.
		49.	35.	14.	0.	0.	0.	0.	0.	0.
TOTAL EXPOSURES			399.	131.	46.	39.	21.	12.	8.	7.
TOTAL RETIREMENTS			254.	62.	2.	0.	0.	0.	2.	1.

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5
1969	1.	0.	0.	0.	1.	0.	1.	0.	1.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	0.	1.	0.	0.	0.	1.	0.	1.	0.	1.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	0.	0.	1.	0.	0.	0.	1.	0.	1.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	0.	0.	0.	1.	0.	0.	0.	1.	0.	1.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1973	1.	0.	0.	0.	1.	0.	0.	0.	1.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	2.	1.	1.	1.	2.	1.	2.	2.	3.	2.
TOTAL RET.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

Table 16. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA PAGE 3 OF 5
UTIL-CAH

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	17.5	18.5	19.5	20.5	21.5	22.5	23.5	24.5	25.5	26.5
1969	0.	0.	1.	3.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	0.	0.	0.	1.	3.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	1.	0.	0.	0.	1.	3.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.
1972	0.	1.	0.	0.	0.	1.	2.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1973	1.	0.	1.	0.	0.	0.	1.	2.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	2.	1.	2.	4.	4.	4.	3.	2.	0.	0.
TOTAL RET.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	27.5	28.5	29.5	30.5	31.5	32.5	33.5	34.5	35.5	36.5
1969	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1973	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL RET.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

Table 16. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA PAGE 5 OF 5

UTIL-CAH

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	37.5	38.5
1969	0.	0.
	0.	0.
1970	0.	0.
	0.	0.
1971	0.	0.
	0.	0.
1972	0.	0.
	0.	0.
1973	0.	0.
	0.	0.
TOTAL EXP.	0.	0.
TOTAL RET.	0.	0.

Table 17. Actuarial trend analysis of Utility Coal and Ash Handlers

IOWA STATE UNIVERSITY
ACTUARIAL TREND ANALYSIS
UTIL-CAH

RETIREMENT BAND	INDICATED AVERAGE LIFE FIRST DEGREE	SECOND DEGREE	THIRD DEGREE	ACTUAL RETIREMENTS FITTED	INDICATED RETIREMENTS FITTED FIRST DEGREE	SECOND DEGREE	THIRD DEGREE
1969-1971	6.4	5.4	22.3	36.	36.	37.	37.
1970-1972	12.6	7.4	23.5	40.	41.	42.	42.
1971-1973	23.9	9.3	22.8	43.	44.	46.	46.
1969-1973	14.7	7.2	23.9	68.	70.	72.	71.
1970-1973	20.5	8.5	26.0	54.	56.	58.	58.
1971-1973	23.9	9.3	22.8	43.	44.	46.	46.
1972-1973	29.6	9.1	23.6	32.	34.	35.	35.
1973-1973	48.3	11.5	31.7	14.	15.	16.	16.

Table 18. Historical arrangement of Utility Ironworkers

HISTORICAL ARRANGEMENT OF MORTALITY DATA PAGE 1 OF 5

UTIL-IW

YEAR	PLANT IN SERVICE JANUARY 1	RETIRED DURING YEAR	PLANT ADD. RETIRED FROM THESE ADDITIONS	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM 0.5	1.5	2.5	3.5	4.5	5.5	6.5
1969	4.		62.	4.	0.	0.	0.	0.	0.	0.
		63.	60.	3.	0.	0.	0.	0.	0.	0.
1970	3.		26.	2.	1.	0.	0.	0.	0.	0.
		26.	23.	2.	1.	0.	0.	0.	0.	0.
1971	3.		18.	3.	0.	0.	0.	0.	0.	0.
		16.	13.	3.	0.	0.	0.	0.	0.	0.
1972	5.		54.	5.	0.	0.	0.	0.	0.	0.
		51.	50.	1.	0.	0.	0.	0.	0.	0.
1973	3.		30.	4.	4.	0.	0.	0.	0.	0.
		22.	20.	2.	0.	0.	0.	0.	0.	0.
TOTAL EXPOSURES			190.	18.	5.	0.	0.	0.	0.	0.
TOTAL RETIREMENTS			166.	11.	1.	0.	0.	0.	0.	0.

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5
1969	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1973	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL RET.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

Table 18. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA

UTIL-IW

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	17.5	18.5	19.5	20.5	21.5	22.5	23.5	24.5	25.5	26.5
1969	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1973	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL RET.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	27.5	28.5	29.5	30.5	31.5	32.5	33.5	34.5	35.5	36.5
1969	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1973	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL RET.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

Table 18. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA

UTIL-IW

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	37.5	38.5
1969	0.	0.
	0.	0.
1970	0.	0.
	0.	0.
1971	0.	0.
	0.	0.
1972	0.	0.
	0.	0.
1973	0.	0.
	0.	0.
TOTAL EXP.	0.	0.
TOTAL RET.	0.	0.

Table 19. Actuarial trend analysis of Utility Ironworkers

IOWA STATE UNIVERSITY
ACTUARIAL TREND ANALYSIS
UTIL-IW

RETIREMENT BAND	INDICATED AVERAGE LIFE FIRST DEGREE	SECOND DEGREE	THIRD DEGREE	ACTUAL RETIREMENTS FITTED	INDICATED RETIREMENTS FITTED FIRST DEGREE	SECOND DEGREE	THIRD DEGREE
1969-1971	0.8	0.8	1.0	9.	9.	9.	9.
1970-1972	1.3	1.2	1.4	7.	7.	7.	7.
1971-1973	50.1	0.6	67.5	6.	6.	129.	6.
1969-1973	30.8	47.8	62.7	12.	12.	12.	12.
1970-1973	36.6	49.3	0.6	9.	9.	9.	9.
1971-1973	50.1	0.6	67.5	6.	6.	128.	6.
1972-1973	83.7	106.4	106.7	3.	3.	3.	3.
1973-1973	50.1	68.0	76.4	2.	2.	2.	2.

Table 20. Historical arrangement of Utility Laborers

HISTORICAL ARRANGEMENT OF MORTALITY DATA PAGE 1 OF 5

UTIL-LAB

YEAR	PLANT IN SERVICE JANUARY 1	RETIRED DURING YEAR	PLANT ADD. RETIRED FROM THESE ADDITIONS	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM 0.5	1.5	2.5	3.5	4.5	5.5	6.5
1969	69.		581.	45.	17.	5.	2.	6.	2.	1.
		594.	545.	37.	9.	0.	0.	1.	0.	0.
1970	76.		385.	36.	8.	8.	5.	2.	5.	2.
		393.	362.	27.	1.	2.	0.	0.	1.	0.
1971	68.		328.	23.	9.	7.	6.	5.	2.	4.
		316.	302.	11.	1.	1.	0.	0.	0.	1.
1972	80.		348.	26.	12.	8.	6.	6.	5.	2.
		339.	322.	12.	3.	0.	0.	0.	0.	0.
1973	89.		403.	26.	14.	9.	8.	6.	6.	5.
		323.	310.	12.	1.	0.	0.	0.	0.	0.
TOTAL EXPOSURES			2045.	156.	60.	37.	27.	25.	20.	14.
TOTAL RETIREMENTS			1841.	99.	15.	3.	0.	1.	1.	1.

YEAR	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM 7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5
1969	1.	1.	1.	0.	0.	0.	0.	0.	0.	2.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	2.	1.	1.	1.	1.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	3.	2.	1.	1.	1.	1.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1973	2.	3.	2.	1.	1.	1.	1.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	9.	8.	6.	4.	3.	2.	1.	0.	0.	2.
TOTAL RET.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

Table 20. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA PAGE 3 OF 5
UTIL-LAB

	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM									
YEAR	17.5	18.5	19.5	20.5	21.5	22.5	23.5	24.5	25.5	26.5
1969	1.	0.	0.	1.	2.	0.	0.	0.	0.	0.
	0.	0.	0.	1.	1.	0.	0.	0.	0.	0.
1970	2.	1.	0.	0.	0.	1.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	0.	2.	1.	0.	0.	0.	1.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	0.	0.	2.	1.	0.	0.	0.	1.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.
1973	0.	0.	0.	2.	1.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	3.	3.	3.	4.	3.	1.	1.	1.	0.	0.
TOTAL RET.	0.	0.	0.	1.	1.	0.	0.	1.	0.	0.

	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM									
YEAR	27.5	28.5	29.5	30.5	31.5	32.5	33.5	34.5	35.5	36.5
1969	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1970	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1971	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1972	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.
1973	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	0.	0.	0.	0.	0.	0.	1.	1.	1.	1.
TOTAL RET.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.

Table 20. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA PAGE 5 OF 5

UTIL-LAB

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	37.5	38.5	39.5	40.5	41.5	42.5	43.5	44.5
1969	0.	0.	1.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.
1970	0.	0.	0.	1.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.
1971	0.	0.	0.	0.	1.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.
1972	0.	0.	0.	0.	0.	1.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.
1973	0.	0.	0.	0.	0.	0.	1.	0.
	0.	0.	0.	0.	0.	0.	0.	0.
TOTAL EXP.	0.	0.	1.	1.	1.	1.	1.	0.
TOTAL RET.	0.	0.	0.	0.	0.	0.	0.	0.

Table 21. Actuarial trend analysis of Utility Laborers

IOWA STATE UNIVERSITY
ACTUARIAL TREND ANALYSIS
UTIL-LAB

RETIREMENT BAND	INDICATED AVERAGE LIFE FIRST DEGREE	SECOND DEGREE	THIRD DEGREE	ACTUAL RETIREMENTS FITTED	INDICATED RETIREMENTS FITTED FIRST DEGREE	SECOND DEGREE	THIRD DEGREE
1969-1971	1.9	2.1	2.0	94.	96.	97.	97.
1970-1972	4.1	4.7	4.0	62.	63.	65.	65.
1971-1973	12.4	9.1	6.4	44.	45.	46.	48.
1969-1973	3.5	3.8	3.2	124.	127.	129.	130.
1970-1973	7.3	6.2	4.8	75.	77.	79.	80.
1971-1973	12.4	9.1	6.4	44.	45.	46.	48.
1972-1973	11.6	9.6	6.3	30.	30.	31.	33.
1973-1973	38.1	13.5	7.9	13.	14.	14.	15.

Table 22. Historical arrangement of Utility Union Employees

HISTORICAL ARRANGEMENT OF MORTALITY DATA

UTIL-UNI

YEAR	PLANT IN SERVICE JANUARY 1	RETIRED DURING YEAR	PLANT ADD. RETIRED FROM THESE ADDITIONS	SURVIVING PLANT BY AGE JANUARY 1 RETIREMENTS THERE FROM 0.5	1.5	2.5	3.5	4.5	5.5	6.5
1969	3306.		1325.	258.	177.	138.	128.	141.	110.	89.
		1575.	1172.	132.	40.	14.	15.	4.	9.	10.
1970	3056.		1141.	153.	126.	137.	124.	113.	137.	101.
		1184.	937.	83.	15.	15.	8.	3.	5.	3.
1971	3013.		903.	204.	70.	111.	122.	116.	110.	132.
		875.	706.	49.	4.	5.	7.	5.	3.	4.
1972	3041.		1210.	197.	155.	66.	106.	115.	111.	107.
		1021.	830.	71.	14.	2.	4.	2.	5.	2.
1973	3230.		1247.	380.	126.	141.	64.	102.	113.	106.
		1117.	831.	202.	12.	7.	2.	1.	1.	2.
TOTAL EXPOSURES			5826.	1192.	654.	593.	544.	587.	581.	535.
TOTAL RETIREMENTS			4476.	537.	85.	43.	36.	15.	23.	21.

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5
1969	70.	78.	92.	65.	106.	110.	89.	85.	176.	153.
	4.	3.	4.	3.	5.	12.	5.	6.	11.	9.
1970	79.	66.	75.	88.	62.	101.	98.	84.	79.	165.
	2.	1.	1.	1.	3.	2.	3.	3.	2.	2.
1971	98.	77.	65.	74.	87.	59.	99.	95.	81.	77.
	2.	3.	2.	0.	2.	3.	3.	3.	2.	2.
1972	128.	96.	74.	63.	74.	85.	56.	96.	92.	79.
	4.	1.	0.	2.	3.	1.	0.	2.	1.	2.
1973	105.	124.	95.	74.	61.	71.	84.	56.	94.	91.
	1.	1.	1.	1.	1.	1.	1.	2.	1.	2.
TOTAL EXP.	480.	441.	401.	364.	390.	426.	426.	416.	522.	565.
TOTAL RET.	13.	9.	8.	7.	14.	19.	12.	16.	17.	17.

Table 22. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA
UTIL-UNI

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	17.5	18.5	19.5	20.5	21.5	22.5	23.5	24.5	25.5	26.5
1969	150.	95.	103.	167.	196.	112.	80.	26.	23.	31.
	9.	4.	9.	10.	15.	8.	8.	2.	0.	3.
1970	144.	141.	91.	94.	157.	181.	104.	72.	24.	23.
	4.	3.	3.	5.	5.	9.	8.	4.	7.	3.
1971	163.	140.	138.	88.	89.	152.	172.	96.	68.	17.
	6.	0.	5.	3.	1.	9.	6.	5.	5.	3.
1972	75.	157.	140.	133.	85.	88.	143.	166.	91.	63.
	2.	1.	3.	7.	0.	3.	4.	12.	7.	5.
1973	77.	73.	156.	137.	126.	85.	85.	139.	154.	84.
	0.	1.	6.	2.	2.	0.	3.	3.	3.	0.
TOTAL EXP.	609.	606.	628.	619.	653.	618.	584.	499.	360.	218.
TOTAL RET.	21.	9.	26.	27.	23.	29.	29.	26.	22.	14.

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	27.5	28.5	29.5	30.5	31.5	32.5	33.5	34.5	35.5	36.5
1969	19.	7.	9.	8.	29.	32.	26.	19.	11.	4.
	2.	1.	1.	1.	3.	4.	5.	4.	3.	0.
1970	28.	17.	6.	8.	7.	26.	28.	21.	15.	8.
	2.	3.	0.	0.	0.	2.	2.	7.	2.	4.
1971	20.	26.	14.	6.	8.	7.	24.	26.	14.	13.
	1.	1.	0.	0.	1.	2.	7.	1.	1.	2.
1972	14.	19.	25.	14.	6.	7.	5.	17.	25.	13.
	1.	1.	3.	0.	0.	1.	1.	1.	2.	4.
1973	58.	13.	18.	22.	14.	6.	6.	4.	16.	23.
	0.	2.	1.	3.	2.	0.	0.	0.	4.	4.
TOTAL EXP.	139.	82.	72.	58.	64.	78.	89.	87.	81.	61.
TOTAL RET.	6.	8.	5.	4.	6.	9.	15.	13.	12.	14.

Table 22. (Continued)

HISTORICAL ARRANGEMENT OF MORTALITY DATA PAGE 5 OF 6
UTIL-UNI

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	37.5	38.5	39.5	40.5	41.5	42.5	43.5	44.5	45.5	46.5
1969	5.	16.	18.	11.	21.	7.	2.	5.	5.	1.
	2.	4.	5.	2.	4.	1.	1.	1.	2.	1.
1970	4.	3.	12.	13.	9.	17.	6.	1.	4.	3.
	0.	0.	4.	6.	2.	3.	1.	0.	4.	1.
1971	4.	4.	3.	8.	7.	7.	14.	5.	1.	0.
	1.	0.	1.	2.	0.	2.	3.	0.	0.	0.
1972	11.	3.	4.	2.	6.	7.	5.	11.	5.	1.
	1.	1.	1.	1.	2.	3.	3.	2.	2.	1.
1973	9.	10.	2.	3.	1.	4.	4.	2.	9.	3.
	0.	0.	0.	0.	0.	2.	1.	0.	6.	2.
TOTAL EXP.	33.	36.	39.	37.	44.	42.	31.	24.	24.	8.
TOTAL RET.	4.	5.	11.	11.	8.	11.	9.	3.	14.	5.

SURVIVING PLANT BY AGE JANUARY 1
RETIREMENTS THERE FROM

YEAR	47.5	48.5	49.5	50.5
1969	2.	0.	1.	0.
	1.	0.	1.	0.
1970	0.	1.	0.	0.
	0.	1.	0.	0.
1971	2.	0.	0.	0.
	2.	0.	0.	0.
1972	0.	0.	0.	0.
	0.	0.	0.	0.
1973	0.	0.	0.	0.
	0.	0.	0.	0.
TOTAL EXP.	4.	1.	1.	0.
TOTAL RET.	3.	1.	1.	0.

Table 23. Actuarial trend analysis of Utility Union Employees

IOWA STATE UNIVERSITY
ACTUARIAL TREND ANALYSIS
UTIL-UNI

RETIREMENT BAND	INDICATED AVERAGE LIFE FIRST DEGREE	SECOND DEGREE	THIRD DEGREE	ACTUAL RETIREMENTS FITTED	INDICATED RETIREMENTS FITTED FIRST DEGREE	SECOND DEGREE	THIRD DEGREE
1969-1971	13.8	7.9	7.8	819.	819.	819.	819.
1970-1972	17.1	11.1	11.0	607.	607.	607.	612.
1971-1973	27.5	9.8	9.7	646.	648.	648.	677.
1969-1973	18.5	8.1	8.0	1296.	1296.	1296.	1312.
1970-1973	24.1	9.4	9.2	893.	893.	893.	929.
1971-1973	27.5	9.8	9.7	646.	648.	648.	677.
1972-1973	24.2	8.4	8.2	477.	482.	492.	516.
1973-1973	17.5	6.3	6.3	286.	296.	310.	320.

Table 24. Mature Females Over Age 30, additions, retirements and balances

INPUT DATA
SIMULATED PLANT-RECORD METHOD
IOWA STATE UNIVERSITY

JUNE 28, 1974

MATURE FEMALES, OVER AGE 30 WHEN HIRED.

ACCOUNT CONTROL CARD

ACCOUNT NAME=OFFICE1 EARLIEST ADDITION= 1957 LATEST ADDITION= 1972 EARLIEST BALANCE= 1957

NO. OF TEST POINTS= 16 INTERVAL BETWEEN TEST POINTS= 0 LAST TEST POINT= 1972 SPR METHOD= BAL.

INPUT DATA= ADDIT. AND BAL. LATEST BALANCE= 1972

INPUT METHOD= CARD

PLANT ADDITIONS

YEAR	ADDITIONS	YEAR	ADDITIONS	YEAR	ADDITIONS	YEAR	ADDITIONS
1957	16.	1961	26.	1965	20.	1969	72.
1958	32.	1962	32.	1966	25.	1970	46.
1959	36.	1963	27.	1967	30.	1971	38.
1960	33.	1964	31.	1968	37.	1972	36.

Table 24. (Continued)

PLANT RETIREMENTS

YEAR	RETIREMENTS	YEAR	RETIREMENTS	YEAR	RETIREMENTS	YEAR	RETIREMENTS
1957	3.	1961	21.	1965	18.	1969	26.
1958	11.	1962	16.	1966	12.	1970	31.
1959	14.	1963	15.	1967	20.	1971	16.
1960	11.	1964	23.	1968	20.	1972	21.

PLANT BALANCES

YEAR	BALANCES	YEAR	BALANCES	YEAR	BALANCES	YEAR	BALANCES
1957	13.	1961	88.	1965	126.	1969	212.
1958	34.	1962	104.	1966	139.	1970	227.
1959	56.	1963	116.	1967	149.	1971	249.
1960	83.	1964	124.	1968	166.	1972	264.

Table 25. Mature Females Over Age 30, comparison with Iowa curves

NO. OF TEST POINTS= 16 INTERVAL BETWEEN TEST POINTS= 0 LAST TEST POINT= 1972

SIMULATED BALANCES METHOD

DISPERSION	AVERAGE SERVICE LIFE	SUM OF SQUARES DIFF.	INDEX OF VARIATION	RET. EXP. INDEX
O4	8.4 YRS.	0.2442E 04	91	83.42
O3	6.9 YRS.	0.3430E 04	108	88.45
O2	5.9 YRS.	0.6042E 04	144	96.53
L0	5.7 YRS.	0.7251E 04	158	99.11
L0.5	5.6 YRS.	0.8294E 04	169	99.65
SC	5.7 YRS.	0.8390E 04	170	100.00
L1	5.5 YRS.	0.9374E 04	180	99.98
S-.5	5.5 YRS.	0.9434E 04	180	100.00
R0.5	5.5 YRS.	0.9592E 04	182	100.00
L1.5	5.4 YRS.	0.1023E 05	188	100.00
S0	5.4 YRS.	0.1057E 05	191	100.00
R1	5.5 YRS.	0.1090E 05	194	100.00
L2	5.4 YRS.	0.1113E 05	196	100.00
S0.5	5.4 YRS.	0.1134E 05	198	100.00
R1.5	5.4 YRS.	0.1179E 05	202	100.00
S1	5.3 YRS.	0.1213E 05	204	100.00
R2	5.2 YRS.	0.1271E 05	209	100.00
S1.5	5.3 YRS.	0.1275E 05	210	100.00
L3	5.2 YRS.	0.1280E 05	210	100.00
R2.5	5.2 YRS.	0.1336E 05	215	100.00
S2	5.2 YRS.	0.1339E 05	215	100.00
R3	5.1 YRS.	0.1403E 05	220	100.00
S3	5.1 YRS.	0.1429E 05	222	100.00
L4	5.1 YRS.	0.1436E 05	222	100.00
R4	5.1 YRS.	0.1487E 05	226	100.00
S4	5.0 YRS.	0.1508E 05	228	100.00
L5	5.0 YRS.	0.1513E 05	229	100.00
R5	5.0 YRS.	0.1549E 05	231	100.00
S5	4.9 YRS.	0.1559E 05	232	100.00
S6	4.9 YRS.	0.1597E 05	235	100.00
SQ	5.4 YRS.	0.1633E 05	237	100.00

Table 26. All Home Office Employees, excluding part time, additions, retirements and balances

INPUT DATA
SIMULATED PLANT-RECORD METHOD
IOWA STATE UNIVERSITY

JUNE 28,1974

ALL HOME OFFICE EMPLOYEES, CLERICAL AND COLLEGE. EXCLUDES P.T.

ACCOUNT CONTROL CARD

ACCOUNT NAME= INSUR EARLIEST ADDITION= 1961 LATEST ADDITION= 1972 EARLIEST BALANCE= 1960 LATEST BALANCE= 1972

NO. OF TEST POINTS= 3 INTERVAL BETWEEN TEST POINTS= 0 LAST TEST POINT= 1972 SPR METHOD= BAL.

INPUT DATA= ADDIT. AND BAL. INPUT METHOD= CARD

PLANT ADDITIONS

YEAR	ADDITIONS	YEAR	ADDITIONS	YEAR	ADDITIONS	YEAR	ADDITIONS
1961	284.	1964	379.	1967	325.	1970	499.
1962	327.	1965	383.	1968	424.	1971	378.
1963	375.	1966	321.	1969	527.	1972	365.

Table 26. (Continued)

PLANT RETIREMENTS

YEAR	RETIREMENTS	YEAR	RETIREMENTS	YEAR	RETIREMENTS	YEAR	RETIREMENTS
*****	0.	1963	274.	1966	378.	1969	406.
1961	314.	1964	303.	1967	331.	1970	392.
1962	296.	1965	371.	1968	335.	1971	342.

YEAR	RETIREMENTS
1972	336.
0	0.
0	0.

PLANT BALANCES

YEAR	BALANCES	YEAR	BALANCES	YEAR	BALANCES	YEAR	BALANCES
*****	971.	1963	1073.	1966	1104.	1969	1308.
1961	941.	1964	1149.	1967	1098.	1970	1415.
1962	972.	1965	1161.	1968	1187.	1971	1451.

YEAR	BALANCES
1972	1480.
0	0.
0	0.

Table 27. All Home Office Employees, excluding part time, comparison with Iowa curves

NO. OF TEST POINTS= 8 INTERVAL BETWEEN TEST POINTS= 0 LAST TEST POINT= 1972

SIMULATED BALANCES METHOD

DISPERSION	AVERAGE SERVICE LIFE	SUM OF SQUARES DIFF.	INDEX OF VARIATION	RET. EXP. INDEX
R1	3.2 YRS.	0.1802E 05	37	100.00
R0.5	3.3 YRS.	0.1925E 05	38	100.00
R1.5	3.2 YRS.	0.1955E 05	38	100.00
S-.5	3.3 YRS.	0.2051E 05	39	100.00
S0	3.2 YRS.	0.2098E 05	40	100.00
SC	3.3 YRS.	0.2109E 05	40	100.00
R2	3.2 YRS.	0.2115E 05	40	100.00
S0.5	3.2 YRS.	0.2165E 05	40	100.00
S1	3.2 YRS.	0.2288E 05	41	100.00
R2.5	3.2 YRS.	0.2311E 05	42	100.00
L0.5	3.3 YRS.	0.2398E 05	42	100.00
L1	3.3 YRS.	0.2413E 05	43	100.00
S1.5	3.2 YRS.	0.2441E 05	43	100.00
L0	3.3 YRS.	0.2463E 05	43	99.99
L1.5	3.2 YRS.	0.2490E 05	43	100.00
R3	3.2 YRS.	0.2519E 05	43	100.00
C2	3.4 YRS.	0.2567E 05	44	100.00
S2	3.2 YRS.	0.2575E 05	44	100.00
L2	3.2 YRS.	0.2634E 05	44	100.00
L3	3.2 YRS.	0.2816E 05	46	100.00
S3	3.2 YRS.	0.2865E 05	46	100.00
R4	3.2 YRS.	0.2930E 05	47	100.00
L4	3.2 YRS.	0.3072E 05	48	100.00
S4	3.2 YRS.	0.3209E 05	49	100.00
L5	3.2 YRS.	0.3343E 05	50	100.00
R5	3.2 YRS.	0.3352E 05	50	100.00
S5	3.2 YRS.	0.3505E 05	51	100.00
S6	3.3 YRS.	0.3630E 05	52	100.00
C3	3.7 YRS.	0.4727E 05	60	95.46
O4	4.3 YRS.	0.7277E 05	74	90.60
SQ	3.2 YRS.	0.8057E 05	78	100.00

Table 28. All Home Office Employees, including part time, additions, retirements and balances

INPUT DATA
SIMULATED PLANT-RECORD METHOD
IOWA STATE UNIVERSITY

JUNE 28, 1974

ALL HOME OFFICE EMPLOYEES, CLERICAL AND COLLEGE. INCLUDES P.T.

ACCOUNT CONTROL CARD

ACCOUNT NAME= OFFICE1   EARLIEST ADDITION= 1948   LATEST ADDITION= 1972   EARLIEST BALANCE= 1947

NO. OF TEST POINTS= 12   INTERVAL BETWEEN TEST POINTS= 0   LAST TEST POINT= 1972   SPR METHOD= BAL.

INPUT DATA= ADDIT. AND BAL.   LATEST BALANCE= 1972

INPUT METHOD= CARD

PLANT ADDITIONS

YEAR	ADDITIONS	YEAR	ADDITIONS	YEAR	ADDITIONS	YEAR	ADDITIONS
1948	298.	1953	378.	1958	430.	1963	429.
1949	264.	1954	326.	1959	539.	1964	435.
1950	379.	1955	353.	1960	459.	1965	444.
1951	470.	1956	428.	1961	327.	1966	441.
1952	541.	1957	497.	1962	365.	1967	509.
						1968	619.
						1969	745.
						1970	671.
						1971	516.
						1972	478.

Table 28. (Continued)

PLANT RETIREMENTS

YEAR	RETIREMENTS	YEAR	RETIREMENTS	YEAR	RETIREMENTS	YEAR	RETIREMENTS
*****	0.	1953	361.	1959	479.	1965	429.
1948	297.	1954	304.	1960	462.	1966	467.
1949	246.	1955	381.	1961	349.	1967	476.
1950	333.	1956	350.	1962	334.	1968	516.
1951	450.	1957	400.	1963	328.	1969	603.
1952	474.	1958	424.	1964	361.	1970	586.
						1971	485.
						1972	465.
						0	0.
						0	0.
						0	0.
						0	0.

PLANT BALANCES

YEAR	BALANCES	YEAR	BALANCES	YEAR	BALANCES	YEAR	BALANCES
*****	589.	1953	758.	1959	993.	1965	1189.
1948	590.	1954	780.	1960	990.	1966	1163.
1949	608.	1955	752.	1961	968.	1967	1196.
1950	654.	1956	830.	1962	999.	1968	1299.
1951	674.	1957	927.	1963	1100.	1969	1441.
1952	741.	1958	933.	1964	1174.	1970	1526.
						1971	1557.
						1972	1570.
						0	0.
						0	0.
						0	0.
						0	0.

Table 29. All Home Office Employees, including part time, comparison with Iowa curves

NO. OF TEST POINTS= 12 INTERVAL BETWEEN TEST POINTS= 0 LAST TEST POINT= 1972

SIMULATED BALANCES METHOD

DISPERSION	AVERAGE SERVICE LIFE	SUM OF SQUARES DIFF.	INDEX OF VARIATION	RET. EXP. INDEX
O4	2.9 YRS.	0.5873E 05	55	100.00
O3	2.7 YRS.	0.6334E 05	57	100.00
O2	2.6 YRS.	0.8769E 05	67	100.00
L0	2.6 YRS.	0.9765E 05	71	100.00
SC	2.6 YRS.	0.1032E 06	73	100.00
L0.5	2.6 YRS.	0.1067E 06	74	100.00
S-.5	2.6 YRS.	0.1128E 06	76	100.00
R0.5	2.5 YRS.	0.1143E 06	77	100.00
L1	2.5 YRS.	0.1168E 06	77	100.00
S0	2.5 YRS.	0.1245E 06	80	100.00
L1.5	2.6 YRS.	0.1264E 06	81	100.00
R1	2.6 YRS.	0.1269E 06	81	100.00
S0.5	2.6 YRS.	0.1336E 06	83	100.00
L2	2.5 YRS.	0.1368E 06	84	100.00
R1.5	2.5 YRS.	0.1376E 06	84	100.00
S1	2.5 YRS.	0.1434E 06	86	100.00
R2	2.5 YRS.	0.1497E 06	88	100.00
S1.5	2.6 YRS.	0.1526E 06	89	100.00
L3	2.5 YRS.	0.1584E 06	90	100.00
R2.5	2.5 YRS.	0.1598E 06	91	100.00
S2	2.5 YRS.	0.1617E 06	91	100.00
R3	2.5 YRS.	0.1707E 06	94	100.00
L4	2.5 YRS.	0.1774E 06	96	100.00
S3	2.5 YRS.	0.1774E 06	96	100.00
R4	2.5 YRS.	0.1857E 06	98	100.00
L5	2.5 YRS.	0.1891E 06	99	100.00
S4	2.5 YRS.	0.1902E 06	99	100.00
R5	2.5 YRS.	0.1925E 06	100	100.00
S5	2.5 YRS.	0.1930E 06	100	100.00
S6	2.5 YRS.	0.1930E 06	100	100.00
SQ	2.5 YRS.	0.1067E 07	235	100.00

Table 30. Manufacturing Unit Hourly, additions, retirements and balances

INPUT DATA
SIMULATED PLANT-RECORD METHOD
IOWA STATE UNIVERSITY

JUNE 26, 1974

MANUFACTURING UNIT HOURLY

ACCOUNT CONTROL CARD

ACCOUNT NAME=MFG1   EARLIEST ADDITION= 1966   LATEST ADDITION= 1973   EARLIEST BALANCE= 1965

NO. OF TEST POINTS= 4   INTERVAL BETWEEN TEST POINTS= 1   LAST TEST POINT= 1973   SPR METHOD= BAL.

INPUT DATA= ADDIT. AND BAL.   LATEST BALANCE= 1973

INPUT METHOD= CARD

PLANT ADDITIONS

YEAR	ADDITIONS	YEAR	ADDITIONS	YEAR	ADDITIONS	YEAR	ADDITIONS
1966	775.	1968	592.	1970	602.	1972	531.
1967	906.	1969	641.	1971	283.	1973	692.

Table 30. (Continued)

PLANT RETIREMENTS

YEAR	RETIREMENTS	YEAR	RETIREMENTS	YEAR	RETIREMENTS	YEAR	RETIREMENTS
*****	0.	1967	953.	1969	615.	1971	324.
1966	[illegible]	1968	673.	1970	338.	1972	625.
						1973	565.
						0	0.

PLANT BALANCES

YEAR	BALANCES	YEAR	BALANCES	YEAR	BALANCES	YEAR	BALANCES
*****	1962.	1967	2494.	1969	2339.	1971	2562.
1966	[illegible]	1968	2318.	1970	[illegible]	1972	2468.
						1973	2591.
						0	0.

Table 31. Manufacturing Unit Hourly, comparison with Iowa curves

NO. OF TEST POINTS= 4 INTERVAL BETWEEN TEST POINTS= 1 LAST TEST POINT= 1973

SIMULATED BALANCES METHOD

DISPERSION	AVERAGE SERVICE LIFE	SUM OF SQUARES DIFF.	INDEX OF VARIATION	RET. EXP. INDEX
L2	4.7 YRS.	0.5869E 06	154	89.91
S1	4.6 YRS.	0.5950E 06	156	94.91
L1.5	4.7 YRS.	0.6007E 06	156	87.62
S1.5	4.6 YRS.	0.6143E 06	157	95.33
S0.5	4.6 YRS.	0.6114E 06	158	91.35
S2	4.5 YRS.	0.6235E 06	159	98.59
L1	4.8 YRS.	0.6255E 06	160	84.12
L3	4.6 YRS.	0.6286E 06	160	95.30
R2	4.6 YRS.	0.6382E 06	161	93.20
S0	4.6 YRS.	0.6399E 06	161	53.71
R2.5	4.5 YRS.	0.6475E 06	162	99.13
R1.5	4.6 YRS.	0.6513E 06	163	95.05
R3	4.5 YRS.	0.6571E 06	165	100.00
R1	4.6 YRS.	0.6806E 06	166	91.73
L0.5	4.9 YRS.	0.6609E 06	166	80.87
S3	4.5 YRS.	0.6890E 06	167	99.65
L4	4.5 YRS.	0.7200E 06	171	99.31
S-.5	4.7 YRS.	0.7208E 06	171	83.57
R4	4.5 YRS.	0.7250E 06	172	100.00
R0.5	4.7 YRS.	0.7403E 06	174	65.41
L0	5.1 YRS.	0.7439E 06	174	77.17
S4	4.5 YRS.	0.7766E 06	176	100.00
L5	4.6 YRS.	0.7991E 06	180	99.96
R5	4.5 YRS.	0.8091E 06	181	100.00
SC	4.8 YRS.	0.8227E 06	183	73.44
O2	5.4 YRS.	0.8330E 06	184	75.45
S5	4.6 YRS.	0.8379E 06	185	100.00
S6	4.6 YRS.	0.8574E 06	187	100.00
O3	7.0 YRS.	0.9296E 06	195	56.94
O4	9.3 YRS.	0.9726E 06	197	60.74
SQ	5.0 YRS.	0.1077E 07	209	100.00

Table 32. Career College Graduates chi-square tests on retirement rate data

Year	Age Int.	Actual Retire.	Expect. Retire.	No. in Orig. Group	Prob. of Retire.	df	calc χ^2	$\chi^2_{.90}$	$\chi^2_{.99}$
1966	0-1	9	11.5	43	.2663	6	4.80	10.6	16.8
	1-2	14	9.6	47	.2040				
	2-3	3	3.6	28	.1296		04, 4 ASL		
	3-4	9	1.6	20	.0801				
	4-5	2	1.1	20	.0532				
	5-6	0	0.4	11	.0387				
1968	0-1	7	7.9	38	.2088	8	8.09	13.4	20.1
	1-2	6	4.6	26	.1772				
	2-3	7	4.1	31	.1312				
	3-4	4	2.4	27	.0902		04, 5.2 ASL		
	4-5	3	1.6	25	.0624				
	5-6	1	0.7	15	.0451				
	6-7	1	0.6	16	.0344				
	7-8	1	0.3	10	.0276				
1970	0-1	9	9.8	36	.2725	10	52.85	16.0	23.2
	1-2	15	8.9	43	.2061				
	2-3	5	3.2	25	.1288				
	3-4	0	1.4	18	.0789		04, 3.9 ASL		
	4-5	2	1.0	19	.0524				
	5-6	4	0.8	21	.0382				
	6-7	4	0.7	22	.0302				
	7-8	2	0.4	14	.0254				
	8-9	1	0.3	13	.0224				
	9-10	1	0.2	9	.0205				

Table 33. Career College Graduates chi-square tests on original group data

Year	Age Int.	Actual Retire.	Expect. Retire.	No. in Orig. Group	Prob. of Retire.	df	calc χ^2	$\chi^2_{.90}$	$\chi^2_{.99}$
1961	0-1	12	10.1	91	.1111	9	11.71	14.7	23.2
-63	1-2	14	9.6		.1055				
	2-3	6	8.8		.0966				
	3-4	7	7.7		.0849		04, 10 ASL		
	4-5	8	6.6		.0720				
	5-6	3	5.4		.0596				
	6-7	2	4.5		.0489				
	7-8	5	3.7		.0401				
	8-11	2	7.7		.0846				
	11 up	32	27.0		.2967				
1964	0-1	17	19.2	139	.1382	6	12.62	10.6	16.8
-63	1-2	30	17.8		.1281				
	2-3	16	15.6		.1119				
	3-4	9	12.8		.0921		04, 8 ASL		
	4-5	7	10.1		.0728				
	5-6	4	7.9		.0567				
	6 up	56	55.7		.4005				
1967	0-1	19	16.4	119	.1382	3	12.21	6.2	11.3
-69	1-2	27	15.2		.1281				
	2-3	9	13.3		.1119		04, 8 ASL		
	3 up	64	74.0		.6220				

Table 34. Manufacturing Marketing chi-square tests on retirement rate data

Year	Age Int.	Actual Retire.	Expect. Retire.	No. in Orig. Group	Prob. of Retire.	df	calc χ^2	$\chi^2_{.90}$	$\chi^2_{.99}$
1966	0-1	1	2.4	21	.1145	22	30.63	30.8	40.3
	1-2	2	2.6	24	.1084				
	2-3	3	1.3	13	.0987				
	3-4	1	1.5	17	.0861				
	4-5	0	0.8	11	.0724		04, 9.7 ASL		
	5-6	2	0.8	13	.0594				
	6-7	2	0.7	14	.0484				
	7-8	0	0.5	13	.0395				
	8-9	0	0.2	6	.0326				
	9-10	0	0.2	6	.0273				
	10-11	0	0.3	13	.0231				
	11-12	1	0.1	5	.0200				
	12-13	0	0.1	14	.0175				
	13-14	0	0.2	13	.0155				
	14-15	0	0.1	6	.0140				
	15-16	0	0.1	7	.0128				
	16-17	0	0.1	7	.0118				
	17-18	0	0.2	16	.0109				
	18-19	1	0.1	14	.0103				
	19-20	0	0.1	12	.0097				
	20-21	0	0.1	6	.0092				
	21-26	1	0.1	4	.0337				

Table 34. (Continued)

Year	Age Int.	Actual Retire.	Expect. Retire.	No. in Orig. Group	Prob. of Retire.	df	calc X^2	$X^2_{.90}$	$X^2_{.99}$
1968	0-1	2	2.8	24	.1169	24	17.88	33.2	43.0
	1-2	7	3.0	27	.1104				
	2-3	2	2.1	21	.1002				
	3-4	4	2.1	24	.0869		04, 9.5 ASL		
	4-5	1	0.9	13	.0726				
	5-6	1	1.0	17	.0593				
	6-7	0	0.5	11	.0480				
	7-8	0	0.5	13	.0391				
	8-9	1	0.5	14	.0322				
	9-10	0	0.4	13	.0269				
	10-11	0	0.1	6	.0228				
	11-12	0	0.1	6	.0197				
	12-13	0	0.2	13	.0173				
	13-14	0	0.1	5	.0154				
	14-15	0	0.2	14	.0139				
	15-16	0	0.2	13	.0127				
	16-17	0	0.1	6	.0117				
	17-18	0	0.1	7	.0109				
	18-19	0	0.1	7	.0102				
	19-20	0	0.2	16	.0097				
	20-21	1	0.1	14	.0092				
	21-22	0	0.1	12	.0089				
	22-23	0	0.1	6	.0086				
	23-27	0	0.1	4	.0320				

Table 34. (Continued)

Year	Age Int.	Actual Retire.	Expect. Retire.	No. in Orig. Group	Prob. of Retire.	df	calc X^2	$X^2_{.90}$	$X^2_{.99}$
1972	0-1	1	3.4	35	.0960	28	39.51	37.9	48.3
	1-2	6	2.1	23	.0922				
	2-3	2	3.0	35	.0863				
	3-4	4	3.1	40	.0783		04,	11.6	ASL
	4-5	0	1.7	24	.0690				
	5-6	1	1.6	27	.0594				
	6-7	2	1.1	21	.0503				
	7-8	0	1.0	24	.0424				
	8-9	0	0.5	13	.0357				
	9-10	1	0.5	17	.0303				
	10-11	1	0.3	11	.0259				
	11-12	0	0.3	13	.0223				
	12-13	0	0.3	14	.0195				
	13-14	0	0.2	13	.0172				
	14-15	0	0.1	6	.0153				
	15-16	0	0.1	6	.0138				
	16-17	1	0.2	13	.0126				
	17-18	0	0.1	5	.0115				
	18-19	0	0.2	14	.0107				
	19-20	0	0.1	13	.0100				
	20-21	0	0.1	6	.0094				
	21-22	0	0.1	7	.0089				
	22-23	0	0.1	7	.0084				
	23-24	0	0.1	16	.0080				
	24-25	0	0.1	14	.0077				
	25-26	0	0.1	12	.0074				
	26-27	0	0.0	6	.0072				
	27-31	1	0.1	4	.0138				

Table 34. (Continued)

Year	Age Int.	Actual Retire.	Expect. Retire.	No. in Orig. Group	Prob. of Retire.	df	calc χ^2	$\chi^2_{.90}$	$\chi^2_{.99}$
1970	0-1	1	3.4	35	.0960	26	34.34	35.6	45.6
	1-2	4	3.7	40	.0922				
	2-3	8	2.1	24	.0863				
	3-4	1	2.1	27	.0783		04, 11.6 ASL		
	4-5	1	1.5	21	.0690				
	5-6	1	1.4	24	.0594				
	6-7	0	0.7	13	.0503				
	7-8	1	0.7	17	.0424				
	8-9	0	0.4	11	.0357				
	9-10	1	0.4	13	.0303				
	10-11	1	0.4	14	.0259				
	11-12	0	0.3	13	.0223				
	12-13	0	0.1	6	.0195				
	13-14	1	0.1	6	.0172				
	14-15	0	0.2	13	.0153				
	15-16	0	0.1	5	.0138				
	16-17	0	0.2	14	.0126				
	17-18	0	0.2	13	.0115				
	18-19	0	0.1	6	.0107				
	19-20	0	0.1	7	.0100				
	20-21	0	0.1	7	.0094				
	21-22	0	0.1	16	.0089				
	22-23	0	0.1	14	.0084				
	23-24	0	0.1	12	.0080				
	24-25	0	0.0	6	.0077				
	25-29	0	0.1	4	.0286				

Table 35. Manufacturing Marketing chi-square tests on original group data

Year	Age Int.	Actual Retire.	Expect. Retire.	No. in Orig. Group	Prob. of Retire.	df	calc χ^2	$\chi^2_{.90}$	$\chi^2_{.99}$
1941	0-1	6	2.3	66	.0342	23	21.54	16.0	41.6
-51	1-2	4	2.2		.0336				
	2-3	2	2.2		.0333				
	3-4	2	2.2		.0328		04, 33 ASL		
	4-5	1	2.1		.0323				
	5-6	1	2.1		.0316				
	6-7	2	2.0		.0308				
	7-8	0	2.0		.0300				
	8-9	2	1.9		.0290				
	9-10	3	1.9		.0280				
	10-11	2	1.8		.0269				
	11-12	1	1.7		.0258				
	12-13	3	1.6		.0246				
	13-14	0	1.5		.0234				
	14-15	0	1.5		.0222				
	15-16	2	1.4		.0210				
	16-17	0	1.3		.0199				
	17-18	0	1.2		.0187				
	18-19	1	1.2		.0176				
	19-20	0	1.1		.0166				
	20-21	1	1.0		.0156				
	21-22	0	1.0		.0147				
	22-23	1	0.9		.0138				
	23 up	32	27.9		.4228				

Table 35. (Continued)

Year	Age Int.	Actual Retire.	Expect. Retire.	No. in Orig. Group	Prob. of Retire.	df	calc χ^2	$\chi^2_{.90}$	$\chi^2_{.99}$
1952	0-1	1	8.0	114	.0699	12	17.14	18.5	26.2
-62	1-2	5	7.8		.0682				
	2-3	14	7.5		.0657				
	3-4	9	7.1		.0624		04,	16 ASL	
	4-5	7	6.6		.0583				
	5-6	7	6.1		.0536				
	6-7	4	5.5		.0485				
	7-8	5	5.0		.0435				
	8-9	5	4.4		.0386				
	9-10	2	3.9		.0341				
	10-11	5	3.4		.0301				
	11-12	1	3.0		.0266				
	12 up	49	45.7		.4005				
1961	0-1	2	5.0	54	.0929	10	14.33	16.0	23.2
-64	1-2	4	4.8		.0894				
	2-3	9	4.5		.0839				
	3-4	7	4.1		.0767		04,	12 ASL	
	4-5	1	3.7		.0682				
	5-6	4	3.2		.0591				
	6-7	0	2.7		.0505				
	7-8	2	2.3		.0428				
	8-9	2	2.0		.0362				
	9-10	3	1.7		.0308				
	10 up	20	20.0		.3697				

Table 35. (Continued)

Year	Age Int.	Actual Retire.	Expect. Retire.	No. in Orig. Group	Prob. of Retire.	df	calc χ^2	$\chi^2_{.90}$	$\chi^2_{.99}$
1964	0-1	1	9.5	85	.1111	7	19.72	12.0	18.5
-67	1-2	11	9.0		.1055				
	2-3	11	8.2		.0966				
	3-4	12	7.2		.0849		04,	10 ASL	
	4-5	12	6.1		.0720				
	5-6	3	5.1		.0596				
	6-7	5	4.2		.0489				
	7 up	30	35.8		.4215				
1967	0-1	4	11.7	126	.0929	4	8.37	7.7	13.3
-70	1-2	15	11.3		.0894				
	2-3	15	10.6		.0839		04,	12 ASL	
	3-4	11	9.7		.0767				
	4 up	81	82.8		.6575				

Table 36. Utility Union Employees, chi-square tests on retirement rate data

Year	Age Int.	Actual Retire.	Expect. Retire.	No. in Orig. Group	Prob. of Retire.	df	calc χ^2	$\chi^2_{.90}$	$\chi^2_{.99}$
1971	0-1	706	71.6	903	.0792	3	6222	6.2	11.3
	1-2	49	87.7	1141	.0768				
	2-3	4	97.4	1325	.0734		04,	14.1	ASL
1973	0-1	831	217.1	1247	.1741	5	2402	9.2	15.1
	1-2	202	187.6	1210	.1550				
	2-3	12	113.1	903	.1252				
	3-4	7	107.1	1141	.0938		04,	6.3	ASL
	4-5	2	90.7	1325	.0684				

Table 37. Utility Union Employees, chi-square tests on original group data

Year	Age Int.	Actual Retire.	Expect. Retire.	No. in Orig. Group	Prob. of Retire.	df	calc χ^2	$\chi^2_{.90}$	$\chi^2_{.99}$
1969	0-1	1172	623.3	1325	.4703	5	926	9.2	15.1
	1-2	83	277.9		.2097				
	2-3	4	121.8		.0919				
	3-4	2	74.2		.0560		04, 2.0 ASL		
	4-5	2	56.8		.0428				
	5 up	62	171.7		.1296				
1970	0-1	937	536.7	1141	.4703	4	599	4.6	9.2
	1-2	49	239.3		.2097				
	2-3	14	104.9		.0919				
	3-4	7	63.9		.0560		04, 2.0 ASL		
	4 up	134	196.8		.1724				